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Dynamic Oligopoly, Investment in Capacity

and Government Firms

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Abstract

This paper examines the role a government firm can play in coordinating the capacity investment decisions of firms in an oligopoly. If the government firm can monitor the investment decisions of all private firms then there exists a reaction function for the government, giving its own investment as a function of all private firms' investment, such that the dominant strategy for any private firm is to choose the socially correct capacity expansion plan. If the government firm can only monitor levels of capital stock complete intertemporal optimality cannot be achieved. However, we show there exists a time-independent reaction function for the government firm, such that private firms playing a non-cooperative differential game using either open-loop or closed-loop Nash strategies will realize steady-state capital stocks that are identical to the socially optimal steady-state capital stocks.

1. Introduction

In this paper we will show how a government firm can be used to regulate the distribution and level of production and investment in an oligopolistic industry. Government ownership and operation of a commercial enterprise which competes with privately owned firms in an oligopolistic industry may be preferred to control via nationalization of the entire industry or to regulation of the industry via taxes and subsidies. Firstly, if a government firm can obtain its desired distribution of production and investment through ownership of a fraction of the industry, this approach can be financially and politically less costly to the government than complete nationalization. Secondly, it is difficult to regulate an oligopolistic industry effectively via taxes and subsidies. Taxation and subsidization of oligopolistic firms may not achieve what was intended, because their reaction to changes in relative prices is indeterminate.[1] Furthermore, for any regulation scheme to be effective, it is necessary to obtain adequate data on cost and demand conditions and to be able to react to changes in these conditions. A government firm can obtain this information in its day-to-day activities and it can respond to these changes without going through the legal or legislative process.

In Harris and Wiens (1977) we demonstrated that a government firm can[2] obtain a distribution of production across firms in an oligopolistic industry, which maximizes social welfare by announcing a suitable reaction of its level of production to the privately owned firms' levels of production. In this paper we will show that a government firm can also obtain a distribution of productive capacity which maximizes social welfare. If the government firm has information on the private firms' flow of investment, it can obtain an optimal distribution of capital both in the short-run and in the long run by announcing a suitable reaction of its rate of investment to the privately owned firms' rate of investment. If the government firm only has information on the distribution of capital at each point of time, we show that the government firm can obtain a long run optimal distribution of capital although inefficiencies may occur in the short-run.

The paper proceeds as follows. In section 2 we consider the static problem and show that a government firm can obtain an optimal distribution of production. Sections 3 and 4 consider the dynamic problem. In section 3 we show that an optimal path of investment for the industry can be obtained if the government firm reacts to the private firms' rates of investment. In section 4 we show that if the government firm monitors capital stock or productive capacity, it can obtain an optimal long run steady-state distribution of productive capacity. A discussion of our results is given in section 5.

2. Static Problem: Monitoring Production

Consider an oligopolistic industry where all firms produce the same homogeneous good. The industry consists of $n + 1$ firms indexed $i = 0, 1, \ldots, n$ with cost functions $C_i(q_i)$ which are convex, increasing with respect to output, q_i, and twice continuously differentiable. The inverse demand function for the industry is given by $D(Q)$ where D is monotonically decreasing and $Q = \sum_{i=0}^{n} q_i$. Denote the government firm by the index $i = 0$. Assume it wants to maximize social welfare given by the conventional surplus measure

$$S(q_0, q_1, \ldots, q_n) = \int_0^Q D(\tau)d\tau - \sum_{i=0}^{n} C_i(q_i), \tag{2.1}$$

i.e., consumer surplus plus producer surplus. The private firms want to maximize profits given by

$$\pi_i(q_o, q_i, \ldots, q_n) = q_i\ D(Q) - C_i(q_i), \quad i = 1\ldots, n. \tag{2.2}$$

As each firm controls its own output level, we have a classic oligopoly situation. As in any oligopoly model there is the traditional problem of choosing an appropriate equilibrium concept. For example, the familiar Cournot-Nash equilibrium for the non-cooperative game given by (2.1) and (2.2) is characterized by the conditions

$$D(Q) + q_i\ D'(Q) = C_i'(q_i) \quad i = 1, \ldots, n. \tag{2.3}$$

or marginal revenue equals marginal cost for the private firms, and

$$D(Q) = C_o'(q_o) \tag{2.4}$$

or price equals marginal cost for the government firm.

Call an allocation $q^* = (q_o^*, q_i^*, \ldots, q_n^*)$ optimal if and only if it maximizes (2.1) and therefore if and only if it satisfies

$$Q^* = \sum_{i=0}^{n} q_i^*, \tag{2.5}$$

and

$$D(Q^*) = C_i'(q_i^*), \quad i = 0, 1, \ldots, n. \tag{2.6}$$

i.e., price equals marginal cost for all firms. If the government can set production levels for each firm in the industry and if its objective is to maximize (2.1) then clearly an optimal allocation will obtain.

The question we are considering here is the following: Can the government firm obtain an optimal allocation of production for the industry if it only controls its own level of output? The answer is yes, provided its output is a suitably chosen function of the outputs of the private firms. The government firm is unique in the industry because it behaves with respect to a different set of objectives than other firms and because its financial resources are far greater than those of the private firms. A credible strategy for the government firm would be to announce a reaction function

$$q_0 = \phi^*(q_1, \ldots, q_n) = Q^* - \sum_{i=1}^{n} q_i. \tag{2.7}$$

Thus the government firm declares that it will make up the difference between the optimal level of production for the industry and the private firms' level of production.

Given the reaction function (2.7), a private firm will choose $\hat{q}_i$ such that $C_i'(q) = D(Q^*)$, provided profits at this price and output level are non-negative.

Given the properties of D and C_i the resulting allocation $\hat{q}$ will be the optimal allocation q^*. Note that a private firm i will choose q_i^* independent of the other firms' output levels since the government firm's reaction function ensures that the price $D(Q^*)$ obtains. In effect the government firm's reaction function given by (2.7) negates the interdependence which results from the industry demand function. Because of this property we say that the reaction function (2.7) strongly supports the Pareto optimal allocation q^* and call ϕ^* a strongly supporting reaction function. It is important to note that while the government firm determines the optimal level of production for the industry, profit maximization on the part of the private firms determines the optimal distribution of production across firms.

An obvious criticism of the above strategy announced by the government firm is that it must have sufficient capacity to make the policy announced in the form of (2.8) credible. Such a criticism, however, is in effect a criticism of the static nature of the model. In a dynamic model the critical feature of an oligopolistic market is the manner in which the investment or capacity installation decisions of firms are determined. It is to this problem we now turn.

3. Dynamic Problem: Monitoring Investment

Can the government firm also obtain an optimal distribution of productive capacity in an oligopolistic industry? To bring out the essential features of the problem consider the following model. Suppose that each firm in the industry can increase its existing capital stock $K_i(t)$ at time t by obtaining an amount $I_i(t)$ of investment. Following Eisner and Strotz (1963) let us assume that a function $G_i(I_i)$ represents the $i^{\underline{th}}$ firms cost of gross investment, where $G_i>0$, $G_i''>0$ for all I_i, $G_i'>0$ for $I_i>0$, $G_i'<0$ for $I_i<0$, and $G_i(0)=0$. Thus the cost per unit of gross investment rises with the investment rate. Assuming that capital depreciates at a constant rate $\delta(0<\delta<1)$, each firm's capital accumulation constraint is given by

$$K_i(t) = I_i(t) - \delta K_i(t)$$

$$K_i(0) = \overline{K}_i, \quad i = 0, 1, \ldots, n. \tag{3.1}$$

Let us also assume that each firm's output $q_i(t)$ is in fixed proportion to its capital stock.[3] To simplify further assume that there are no operating costs. By a suitable choice of units the production function can be written as

$$q_i(t) \leq K_i(t), \quad i = 0, 1, \ldots, n. \tag{3.2}$$

In general we shall assume (3.2) holds with equality. Denote the time paths of investment and capital stock, respectively, by

$$\{I(t)\} = \{I_0(t), I_1(t), \ldots, I_n(t)\}, \text{ and}$$

$$\{K(t)\} = \{K_0(t), K_1(t), \ldots, K_n(t)\}.$$

Denote the time dependent inverse demand function by $D[Q(t), t]$ where

$$Q(t) = \sum_{i=0}^{n} q_i(t) = \sum_{i=0}^{n} K_i(t) = K(t).$$

Finally, assume that the government firm's objective function is given by

$$W = \int_0^\infty e^{-rt} S(Q, I_t, t)dt \tag{3.3}$$

where

$$S(Q, I, t) \equiv \int_0^Q D(\tau, t)d\tau - \sum_{i=0}^{n} G_i(I_i)$$

and the private firm's objective function by

$$V_i = \int_0^\infty e^{-rt} [K_i(t) D(Q_t, t) - G_i[I_i(t)]) dt \tag{3.4}$$

Equations (3.3) and (3.4) are the dynamic versions of Equations (2.1) and (2.2). The government firm is concerned with maximizing the discounted present value of producer plus consumer surplus, where the discount rate is the market interest rate. This latter assumption could be modified without much difficulty. W will be referred to as the social welfare function.

After we derive the social welfare optimal trajectories of investment and capital stock we will show that the government firm can obtain these same trajectories by announcing a suitable reaction function. The social optimality conditions are obtained by maximizing equation (3.3) subject to equations (3.1) and (3.2). Denote the optimal time path of investment and capital by $\{I^*(t)\}$ and $\{K^*(t)\}$, and let $P_i(t)$ denote the undiscounted co-state variable attached to the capital stock of the $i^{\underline{th}}$ firm. Given an interior solution for all $t \geq 0$, then the optimal trajectories must satisfy the following necessary conditions:

$$G_i'[I_i^*(t)] = P_i(t), \tag{3.5}$$

$$\dot{P}_i(t) = (r + \delta)P_i(t) - D[K^*(t), t], \tag{3.6}$$

$$\dot{K}_i(t) = I_i^*(t) - \delta K_i^*(t), \quad i = 0, 1, \ldots, n. \tag{3.7}$$

Differentiating (3.5) and using (3.6) we obtain the instantaneous marginal conditions of the rate for change of investment,

$$\dot{I}_i^*(t) = \frac{(r+\delta)\; G_i'(I_i^*(t)) - D(K^*(t), t)}{G_i''(I_i^*(t))} \tag{3.8}$$

$$i = 0, 1, \ldots, n.$$

As the problem is one of maximization over an infinite horizon the solution will depend upon what happens in the limit to the demand function $D(Q,t)$. We shall assume the demand function converges to a stationary demand function, $D^*(Q)$, and that the optimal capital stocks converge to steady state values, $(K_0^*, K_1^*, \cdots, K_n^*)$, where

$$I_i^* = \delta K_i^*, \quad i = 0, 1, \ldots, n, \tag{3.9}$$

and

$$G_i'(I_i^*) = \frac{D^*(K^*)}{r+\delta} \qquad i = 0, 1, \ldots, n. \tag{3.10}$$

Equations (3.7) - (3.10) together with the initial conditions, $K_i^*(0) = \overline{K}_i$ completely determine the social welfare optimal trajectories $\{K^*(t)\}$ $\{I^*(t)\}$.

We will now show that the government firm can in certain circumstances obtain the optimal trajectories of $\{I^*(t)\}$ and $\{K^*(t)\}$ by announcing a suitable reaction function. Let $\{I(t)\}$ be the time path of investment chosen by the private firms. Consider the dynamic reaction function $\phi(I_1, \ldots, I_n, t)$ given by

$$I_0(t) = \phi(I_1, \ldots, I_n, t) = I_0^*(t) + \sum_{i=1}^{n} (I_i^*(t) - I_i) \tag{3.11}$$

In effect the government firm announces that it will make up the difference between desired industry investment and actual investment by the private firms. Notice that this strategy will also ensure that the desired industry capital stock is obtained at each point in time. Since

$$\sum_{i=0}^{n} \dot{K}_i(t) = \sum_{i=0}^{n} I_i(t) - \delta\sum_{i=0}^{n} K_i(t) = \sum_{i=0}^{n} I_i^*(t) - \delta\sum_{i=0}^{n} K_i(t), \tag{3.12}$$

and

$$K_i(0) = K_i^*(0) = \overline{K}_i, \quad i = 0, 1, \ldots, n \tag{3.13}$$

we have

$$K^*(t) = \sum_{i=0}^{n} K_i^*(t) = \sum_{i=0}^{n} K_i(t) \tag{3.14}$$

At this point we are faced with a difficult problem of choosing an appropriate model of a dynamic oligopoly situation. While there has been some progress in this area, the choice of an appropriate equilibrium concept is far from settled. Consider, however, the individual firm's problem given the reaction function (3.11) and the resulting time path of aggregate capacity $\{K^*(t)\}$. This is given by

$$\max_{I_i(t)} \int_0^\infty e^{-rt}[K_i D[Q^*(t), t] - G_i(I_i)]dt \tag{3.15}$$

subject to (i) $\dot{K}_i = I_i - \delta K_i$,

(ii) $K_i(0) = \overline{K}_i$.

As the individual firm cannot affect the time path of $Q^*(t)$ its dominant strategy will be to choose the investment policy $\{I_i^*(t)\}$ and consequently the welfare optimal allocation of investment, both across firms and over time will be realized. This can be seen by solving the control problem (3.15) and noting that the resulting differential equations and intitial conditions are identical to (3.7) through (3.10), for all private firms. The government firm must necessarily follow the optimal policy as $K_0^*(t) = K^*(t) - \sum_{i=1}^{n} K_i^*(t)$. Notice that the effect of the government's reaction function is to eliminate the interdependence among the private firms and the oligopoly problem in this case becomes trivial.

4. Dynamic Problem: Monitoring Capital Stock

Suppose the capital stocks in an industry are easier variables for a government firm to observe than the investment flows. Due to the costs of acquiring information instantaneously this may well be a relevant constraint. It is equivalent to assuming in a discrete time framework that the ex ante investment decisions of the private firms are not monitored, but that the ex post results of these decisions are monitored. Can the government firm obtain a Pareto optimal distribution of capital and investment by announcing its reaction function in terms of the stock variable rather than the flow variable? This procedure may be less costly measured in terms of gathering data as information on the rate of investment is obtained only after the capital has been installed. Because of this lag in information, we would not expect to obtain the optimal distribution of capital and investment at each point of time, i.e., in the short-run. Monitoring capital stock is basically a long run view. The appropriate question is whether the government firm can obtain an optimal steady state distribution of capital across firms.

The government firm's ability to control the industry depends on the information it has available. In the last section we showed that if the government firm obtains information on the flow of investment at each point of time, then it can control the industry by setting the time path of aggregate industry capital (which is equivalent to setting the time path of the price of output for the industry). The solution to each private firm's maximization problem was obtained by solving a conventional optimal control problem. In effect the government firm's reaction function negated the interdependency among the private firms induced by the industry demand function.

The assumption of this section that the government firm obtains information on the stock of capital at each point of time but not on the current flow of investment implies that it may not be able to control the time path of aggregate industry capital in the short-run. As a result its reaction function does not eliminate the interdependence among the private firms.

We will model the behaviour of the private firms in the industry as an n player noncooperative differential game. The government firm seeks to maximize social welfare given by the equation (3.3) by announcing a suitable reaction function,

$$I_0 = \phi(K_1, \dots, K_n)$$

which is time independent. Each private firm seeks to maximize profits given by equation (3.4).

We will present the necessary conditions for a Nash equilibrium trajectory first on the assumption that the private firms play an open-loop noncooperative differential game and second on the assumption that the private firms play a closed-loop noncooperative differential game.[4]

The use of the Nash equilibrium assumption suffers from the usual difficulties encountered in oligopoly theory but for our purposes it will suffice in illustrating the basic role played by the government firm. An <u>open-loop Nash equilibrium</u> is a set of investment strategies $\{I'(t)\}$ such that

$$V_i[\{I'(t) \setminus I_i(t)\}] \leq V_i[\{I'(t)\}] \text{ for all admissable } \{I_i(t)\}, \text{ and all } i=1,\cdots,n,$$

where $\{I'(t) \setminus I_i(t)\}$ denotes that all firms $j \neq i$ follow the strategies $\{I_j'(t)\}$ while the i^{th} firm chooses the admissable strategy $\{I_i(t)\}$.

Note that an open-loop Nash equilibrium requires complete knowledge of opponents' strategies for all future time periods. Strategies are chosen by all firms at time zero and in future time periods firms merely carry out the strategies initially chosen.

Let us assume that the industry demand function approaches a stationary limit demand function, $D(Q)$. Let $K^* = \sum_{i=0}^{n} K_i^*$ where $(K_0^*, K_1^*, \cdots, K_n^*)$ is the distribution of capital stock which obtains in a long run social welfare maximum. Consider the government firm's reaction function given by

$$I_0(t) = r[K^* - K_0^* - \sum_{i=1}^{n} K_i(t)] + \delta K_0(t) \tag{4.2}$$

where $K_i(t)$, $i = 1, \ldots, n$ are the capital stock trajectories chosen by the private firms and $K_0(t)$ is the resulting government firm trajectory.

The necessary conditions for an open-loop Nash equilibrium are given as follows. The Hamiltonian for the private firm, $i = 1, \ldots, n$ is

$$\begin{aligned} H_i = e^{-rt}\{&K_i(t)D(K(t)) - G_i(I_i(t)) + P_i(t)[I_i(t) - \delta K_i(t)] \\ &+ P_0^i(t)[I_0(t) - \delta K_0(t)]\} \end{aligned} \tag{4.3}$$

where $K(t) = \sum_{i=0}^{n} K_i(t)$, and $I_0(t)$ is given by (4.2). Since each individual private firm takes the state variables of the other private firms as given there is no need to include in the $i^{\underline{th}}$ Hamiltonian the differential equations for K_j, $j \neq i, 0$.

An open-loop Nash equilibrium $\{I_i'(t)\}_{i=1}^{n}$ must satisfy the following necessary conditions (we have omitted the time index for convenience):

$$G_i'(I_i') = P_i$$

$$\dot{P}_i = (r + \delta)P_i - D(K) - K_i D'(K) + rP_o^i \qquad (4.4)$$

$$\dot{P}_o^i = rP_i^o - K_i D'(K). \quad i = 1,\ldots, n$$

Assuming the Nash equilibrium converges to a steady state, where the steady state capital stocks are denoted by K_i', $i = 0, 1,\ldots, n$, we get the following conditions, using the steady-state values of (4.4) and (4.2):

$$(1) \quad \sum_{i=1}^{n} K_i' = K^* - K_o^*$$

$$(2) \quad G_i' \, (\delta K_i') = \frac{D(K^*)}{r+\delta} \qquad (4.5)$$

As $G'(\cdot)$ is a monotone increasing function, (4.5) and (3.4) - (3.10) imply $K_i' = K_i^*$ for all $i = 0, 1,\ldots, n$. Thus it is necessarily true that in an open-loop Nash equilibrium, given the reaction function (4.3) for the government firms, private firms will choose long run equilibrium capital stocks identical to the long run social welfare optimal capital stocks. In addition the reaction function also ensures that the government firm will end up with the correct long run level of capital stock.

The concept of an open-loop Nash equilibrium has been criticized as being an inappropriate equilibrium concept for games occuring over many time periods as it does not allow for the possibility that players may want to change their strategies depending upon the evolution of the game.[5] In differential game theory an alternative equilibrium concept which attempts to meet this criticism is the closed-loop Nash equilibrium. Players in the game are assumed to choose strategies which be described as functions of the state variables in the game and time. Let $I_i = \psi_i(K_o, K_1,\ldots, K_n, t)$ denote any such arbitrary strategy which is often referred to as a closed-loop control.

In order to illustrate the concept of a closed-loop Nash equilibrium consider the game in discrete time. Let

$$V_i[\psi,t+1], \ i=1,\cdots,n$$

denote the return to the i^{th} player for all periods from t+1 on, when players use the set of strategies ψ. The transition function from one state to another is given by $K_{t+1} = f(I,K_t,t)$. Let $\hat{\psi}_i$ denote the list of strategy functions with the ith component deleted and similarly $\hat{I}_i$ denotes the list of n-1 controls, with the ith component deleted. Let $\pi_i(\hat{I}_i,I_i,K_t,t)$ denote the current payoff to the i^{th} player at time t if the state of the game is K_t and players choose the vector of controls $I = (\hat{I}_i,I_i)$. If an n-tuple of strategies $\psi^* = (\psi_1^*,\cdots,\psi_n^*)$ are closed-loop Nash equilibrium strategies then

$$\pi_i(\hat{\psi}_i^*,I_i^*,K_t,t) + V_i[\psi^*(K_{t+1}^*),t+1] \geq$$
$$\pi_i(\hat{\psi}_i^*,I_i,K_t,t) + V_i[\psi^*(K_{t+1}),t+1] \quad (4.5)$$

for all I_i, and all $i=1,\cdots,n$, where

$$K_{t+1}^* = f[\psi^*(K_t),K_t,t]$$

and

$$I_i^* = \psi_i^*(K_t,t), \text{ all } i=1,\cdots,n.$$

Intuitively a closed-loop Nash equilibrium can be explained as follows. Suppose a player j knows all other players strategies $\psi_i, i\neq j$, and he is considering following a particular strategy ψ_j^*. Suppose the game is currently in some state at some time period. He then asks himself what would be the optimal current control decision to make given that all other players will play ψ_i for all time periods and he will play $\psi_j{}^*$ for all future time periods. In making this choice he takes into account the effect his current decision will have on the evolution of the state variables of the game. If the optimal control chosen is identical to one given by $\psi_j{}^*$ for that time period, and this is true for all states and all time periods then we say that $\psi_j{}^*$ is optimal against ψ_i, $i \neq j$. A closed-loop Nash equilibrium is then a set of strategies which are all mutually optimal against the others.

Starr and Ho [1969] have given a set of necessary conditions for a set of strategies $\{\psi_1, \ldots, \psi_n\}$ to be a set of closed-loop Nash equilibrium strategies. Define the Hamiltonian for the $i^{\underline{th}}$ player to be

$$H_i = e^{-rt}\{K_i D(K) - G_i(I_i) + \sum_{j=1}^{n} \lambda_{ij}(I_j - \delta K_j) + \lambda_{io}(r[K^* - K_o^* - \sum_{i=1}^{n} K_i(t)])\}$$

where we have substituted the government reaction function (4.2) into the differential equation describing the evolution of the government firm's capital stock. λ_{ij} is a co-state variable for the $i^{\underline{th}}$ firm attached to the $j^{\underline{th}}$ capital stock. Necessary conditions for $(\psi_1, \ldots, \psi_n)$ to be closed-loop Nash equilibrium strategies are that the resulting trajectory of state and co-state variables satisfy

$$\dot{K}_i = I_i - \delta K_i \tag{4.6}$$

$$\dot{K}_o = r[K^* - K_o^* - \sum_{i=1}^{n} K_i\ (t)] \tag{4.7}$$

$$\dot{\lambda}_{ik} = r\,\lambda_{ik} - K_i D'(K) + \delta\lambda_{ik} + r\lambda_{io} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \lambda_{ij} \frac{\partial \psi_j(K,t)}{\partial K_k} \tag{4.8}$$

$$k \in \{1,\cdots,n\},\ k\neq i,$$

$$\dot{\lambda}_{ii} = r\lambda_{ii} - K_i D'(K) - D(K) + \delta\lambda_{ii} + r\lambda_{io} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \lambda_{ij} \frac{\partial \psi_j(K,\ t)}{\partial K_i} \tag{4.9}$$

$$\dot{\lambda}_{io} = r\lambda_{io} - K_i D'(K), \tag{4.10}$$

$I_i^* = \psi_i(K,\ t)$ is the solution to

$$\max_{I_i}\ H_i[K,\ \psi_1,\dots,\ \psi_{i-1},\ I_i,\ \psi_{i+1},\dots,\ \psi_n,\ t], \tag{4.11}$$

The conditions (4.6) through (4.11) hold for all private firms, $i=1,\cdots,n$.

Now assume the closed-loop Nash equilibrium of this differential game converges to a steady-state, with a state vector $(K_o',\dots,\ K_n')$. From (4.7) of the necessary conditions we have that

$$\sum_{i=1}^{n} K_i' = K^* - K_o^* \tag{4.12}$$

As all players are in a steady-state we have that

$$I_j = \psi_j(K_o',\dots,\ K_n') = \delta K_j', \qquad j = 1,\dots,\ n \tag{4.13}$$

where

$$\psi_j(K_o,\dots,\ K_n) = \lim_{t\to\infty} \psi(K_o,\dots,\ K_n,\ t).$$

Consider the matrix of reaction coefficients evaluated at the steady rate

$$R' \equiv \left[\frac{\partial\psi_j}{\partial K_i}\right]_{i,j=1,\ldots,n}$$

where R is an nxn matrix.[7] In a steady state situation it seems natural to make the following assumptions on R

(A.1) $\frac{\partial\psi_j}{\partial K_j} < 0$ for all $j = 1,\ldots, n$.

(A.2) $\left|\frac{\partial\psi_j}{\partial K_j}\right| > \sum_{\substack{i=1 \\ i\neq j}}^{n} \left|\frac{\partial\psi_i}{\partial K_j}\right|$ for all $j = 1,\ldots, n$.

The first assumption simply says that an increase in a firm's own capital stock will cause it to lower its rate of investment. The second assumption states that a firm's own reaction to an increase in its capital stock is greater in magnitude than the sum of all other firms' reactions.

Let λ_i denote the vector $(\lambda_{i1},\ldots, \lambda_{i,i-1}, \lambda_{i,i+1},\ldots, \lambda_n)'$. Using the steady-state values of the co-state equations, (4.8) can be written as

$$[(r + \delta)I - R_i]\ \lambda_i = \underline{0} \tag{4.14}$$

where R_i is the R matrix with the $i^{\underline{th}}$ column and row deleted, and I denotes the (n-1)x(n-1) identity matrix. By (A.1) and (A.2) the matrix $[(r + \delta)I - R_i]$ has the dominant diagonal property with positive diagonal elements, and thus is a matrix of full rank. Consequently the only solution to (4.14) will be $\lambda_i = \underline{0}$ and this will be true for $i = 1,\ldots, n$.

Using the above result the necessary condition (4.9) in steady-state becomes

$$(r + \delta)\ \lambda_{ii} = D^*(K) \tag{4.15}$$

and by (4.15) and (4.11)

$$\lambda_{ii} = G_i' \ [\delta K_i']. \tag{4.16}$$

Putting (4.12)-(4.16) together gives

$$G_i'(\delta K_i') = \frac{D(K^*)}{(r+\delta)}, \qquad i = 1,\ldots, n \tag{4.17}$$

which implies, as $G_i'(\cdot)$ is an increasing function, that $K_i' = K_i^*$, $i = 1,\ldots, n$ and thus $K_0' = K_0^*$. Thus we have shown that any closed-loop Nash equilibrium, whose equilibrium strategy functions satisfy (A.1) and (A.2), given the reaction function (4.2) for the government firm, will give a steady-state distribution of capital stocks across firms identical to the optimal steady-state distribution of capital stocks.

The assumptions (A.1) and (A.2) on the strategy or reaction functions ψ_i do not seem unreasonable. If in a long-run steady-state a firm was suddenly faced with an exogenous increase in its capital stock, all other firms capital stocks constant, it seems natural that the firms would wish to run down its capital stock towards the steady-state optimal value. (A.2) is simply an assumption about the degree to which firms react. We cannot, however, logically exclude the possibility that closed-loop Nash equilibrium exist whose reaction functions do not satisfy (A.1) and (A.2). In such cases we cannot be sure that trajectories with the correct limiting values will be realized.[7]

5. Conclusions

The exercise carried out in this paper is very close in spirit to the second theorem of classical welfare economics, which states that given allocations which are Pareto optimal there exists a set of prices such that competitive maximizing agents when faced with those prices will choose the specified allocation. The questions we posed in this paper is analogous to the question posed by the second theorem. Given an allocation of outputs across firms within an industry, which is optimal relative to a partial equilibrium welfare measure,can some form of "decentralized" control be found to implement that allocation? One aspect of this problem we have chosen to focus upon is that firms in the industry do not behave as price-takers because changes in their individual outputs significantly affect the market price. The industry in question could be described as one consisting of firms which are non-collusive oligopolists. There are, of course, many means by which such some control of the industry can be obtained. The one method we are primarily concerned with is the use of a government firm within the industry. In a previous paper we demonstrated that for the static case, if the government firm announces its strategy in the form of a reaction function, giving its own output level as a function of the output levels of all other firms, then a reaction function exists such that the private firms behaving as Nash oligopolists will achieve an industry allocation of output identical to the optimal allocation.

The main criticism one can make of this result is that the government firm must have sufficient productive capacity to make its reaction function credible. Properly formulated however such a criticism calls for an explicit dynamic analysis of the problem, and in particular the decisions by firms to invest in new capacity. In this paper we carried out such a dynamic analysis and found that the results of the static case more or less carry over to the dynamic case. If the government firm is capable of monitoring the flow of investment by all private firms then a reaction function exists such that the dominant strategy for any private firm is to choose the social welfare optimal investment path. In this case the static results and the dynamic results are identical. We argued, however, that there might well be reasons the government firm could not monitor the investment undertaken by private firms but rather could only monitor the levels of capital stock of all private firms. In this case we showed that the closed and open-loop Nash capital stock trajectories of the differential game would converge to the welfare optimal steady capital stocks. Thus while short run optimality was not feasible, long run optimality could be achieved with a suitable reaction function.

The results of this type of analysis certainly do not carry any immediate policy implications for the operation of government owned firms in oligopolistic industries. Rather it suggests that the reaction function approach is one way a government firm might affect the performance of the industry. Certainly it should be compared with other regulation mechanisms, such as price or quantity regulation, in order to make some judgement about its relative strengths and weaknesses. In making such comparisons however, one should pay attention to those features, such as uncertainty, informational asymmetries, or bureaucracic incentive problems, which should highlight the differences between alternative regulation mechanisms. Much remains to be done.

FOOTNOTES

1. See Guesnerie and Laffont [1976] for a demonstration of the problems encountered when taxing firms with market power.

2. Much of Harris and Wiens [1977] was concerned with oligopoly in an industry producing differentiated products. This paper concentrates on dynamic problems and ignores the difficulties peculiar to product differentiation. An early analysis of the static problem was carried out by Merrill and Schneider [1966].

3. Thus whenever we use the term capital stock it can also be interpreted as productive capacity.

4. For a survey of non-zero sum differential games and the alternative equilibrium concepts proposed see Starr and Ho [1969].

5. See Starr and Ho [1969] for an elaboration of this criticism.

6. R' denotes the transpose of the matrix R.

7. In this model both the closed and open loop equilibrium strategies have trajectories with the same limiting values. In general, however, closed and open loop equilibrium trajectories need not coincide.

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