77-13

# Government Enterprise: An Instrument for the Internal Regulation of Industry

by

Richard G. Harris
Queen's University, Kingston

and

Elmer G. Wiens
Carleton University, Ottawa

CARLETON ECONOMIC PAPERS

# Government Enterprise: An Instrument for the Internal Regulation of Industry*

by

Richard G. Harris
Queen's University, Kingston

and

Elmer G. Wiens
Carleton University, Ottawa

July 1977

*Preliminary draft: This is a revised version of Carleton Economic Paper 77-05. Any comments would be appreciated.

## 1. Introduction

The literature on public control of industry has largely been concerned with providing the rationales for control and analyses of alternative methods of achieving public objectives. Market failure arising from either non-pecuniary externalities or the structural characteristics of an industry is a basic rationale for intervention. The distinction between these types of market failure is well known. Pollution and health hazards, for example, are problems of market failure due to non-pecuniary externalities. Structural characteristics of an industry may induce market failure in the sense that the market power of monopolies, oligopolies and other forms of imperfect competition lead to an inefficient allocation of resources. We wish to concentrate on the analysis of a policy instrument designed to deal with structural market failure. The traditional policy instruments in this area have been antitrust legislation and direct regulation of certain facets of industry behaviour. We will examine government enterprise as an alternative mechanism for control of private markets. In particular, we deal with the situation in which a publicly owned firm competes with privately owned firms in an oligopolistic setting.[1] This situation is quite common in many western economies. For example, in Canada more than ten Crown corporations and numerous provincial government firms fall into this category. In spite of the prevalence of this form of public organization, with the exception of a paper by Merrill and Schneider [1966], we find virtually no economic analysis of this problem.[2] This paper will examine some of the issues associated with government competition and in particular will focus on the role a government firm can play as an instrument of regulation internal to the industry.

We should point out that this paper is not concerned with the positive issues of how public enterprises behave. While there has been some empirical work on this problem by Martin [1959], Sheahan [1960] and Davies [1971] it remains a largely unexplored and important area of research.[3] Rather the analysis focuses on how government enterprise should be used to promote static economic efficiency within a non-competitive market structure. We assume that the set of policy instruments available are limited to those variables under the control of the government owned firm, and the models used are partial equilibrium as they deal with only a single industry within the economy. Consequently, the analysis is an exercise in 'second best' piecemeal welfare economics. Implicit in the analysis of a government firm within an otherwise privately controlled industry is the assumption that complete nationalization of the industry or direct regulation of all facets of industry activity are not viable policy alternatives. If either of these options was available and desirable there would be no particular role for a government firm to play.

Perhaps the most interesting feature of operating a government firm within an oligopolistic industry is that strategic considerations must explicitly be accounted for. The actions of the government firm will affect the actions of other firms in the industry and this is precisely where the public firm has some scope for affecting the performance of the industry. We shall investigate, under the usual assumptions of traditional oligopoly theory, how various actions of the government firm can manipulate the private firms' behaviour to improve industry performance.[4]

A second part of the paper is devoted to considering iterative schemes a government firm might adopt when it has less than complete information. This is an important aspect of the problem, since a policy instrument's effectiveness is limited by the quality of information available to those in command of the instrument.

A final part of the paper is devoted to a brief comparison of public enterprise with alternative policy instruments as a means of improving the performance of oligopolistic industries. The last section of the paper gives some conclusions and suggestions for future research.

## 2. Oligopoly and Government Firm Action

In this section we ask how alternative modes of government firm behaviour can affect the equilibrium allocation of resources in an oligopolistic industry. The analysis is static and partial equilibrium with complete information, and resource allocation is judged in terms of the conventional partial equilibrium welfare measure of producer plus consumer surplus. We assume that all private firms in the industry behave as Cournot-Nash oligopolists, independently of how the government firm acts. This assumption has well known difficulties[5] but for lack of a better alternative we shall use it throughout the paper. The private firms in the industry thus behave as non-collusive oligopolists. The case in which all private firms perfectly collude to maximize their joint profits would be a special case of an industry consisting of two firms, one public and one private.

There is a single government firm in the industry whose objective is to maximize social welfare. All private firms behave as Cournot-Nash oligopolists with respect to the government firm. The government firm controls only its own output level and it can only affect other firms' output choices to the extent that their output choices depend upon the government firm's output choice. There are a number of alternative ways the government firm can act vis-a-vis the other firms in the industry. We find it useful, however, to focus on three different types of action and to compare the outcomes in terms of the resulting resource allocation. These alternative behavioural modes are: a) the Cournot-Nash Mode; b) the Stackleberg Mode; and c) the Reaction Function Mode. Before proceeding with an analysis of these we introduce some notation and the basic model.

The inverse demand function for the market is given by

$$P = D(Q)$$

where Q is aggregate industry output and $D'(Q) < 0$. The industry consists of n+1 firms indexed $i=0,1,\cdots,n$ with cost functions $C_i(q_i)$ which are positive, convex, increasing in output and twice continuously differentiable*. The output of the ith firm is $q_i$. We assume entry to the industry is blocked and consequently n is taken as fixed. By definition $Q = \sum_{i=0}^{n} q_i$. The government firm is indexed $i=0$. Welfare is measured by

$$W(q_0,q_1,\cdots,q_n) \equiv \int_0^Q D(\tau)d\tau - \sum_{i=0}^{n} C_i(q_i), \tag{2.1}$$

and each private firm wishes to maximize its profits given by

$$\pi_i(q_0,\cdots,q_n) \equiv q_i D(Q) - C_i(q_i), \quad i=1,\cdots,n.$$

In the language of non-cooperative game theory W is the payoff function to the government firm, $\pi_i$ are the payoff functions to the private firms, and $q_i$ is the strategy variable for the ith player.

a) <u>Cournot-Nash Mode:</u> Suppose the government firm acts as a Cournot-Nash player in the oligopoly game. Given that all private firms act as Cournot-Nash oligopolists what can be said of the resulting allocation of resources? Let $q^* = (q_0^*,q_1^*,\cdots,q_n^*)$ denote the resulting Nash equilibrium allocation of output across firms. From the individual optimality of the output choices of private firms we get

$$q_i^* D'(Q^*) + D(Q^*) = C_i'(q_i^*), \quad i=1,\cdots,n, \tag{2.2}$$

or marginal revenue equals marginal cost. The government firm's choice of $q_0$, since it assumes that its output choice does not affect the output choices of the private firms, will satisfy

$$D(Q^*) = C_0'(q_0^*). \tag{2.3}$$

Thus the government firm chooses its output such that price equals marginal cost. Note that while this rule is the conventional "first-best" rule of welfare economics the resulting allocation is not "first-best" even in this partial equilibrium context. Maximization of W(·) by a planner with respect to all output variables would imply that price should equal marginal cost for all firms and from (2.1) we see this is not the case.

Consider a situation of duopoly with one private and one public firm. Suppose that both firms have identical cost functions and the cost functions exhibit constant average cost. In this case the only possible equilibrium is with the government firm producing all of the market demand, i.e., $Q = q_0$, and selling it at average cost. Thus a situation starting out as an oligopoly will terminate with a government monopoly. The dynamics of the linear demand case are illustrated in Figure 1. The arrows indicate the dynamics starting from an initial point e. It is interesting that in the identical constant average cost case it is not possible for an equilibrium to occur with both firms sharing the market.

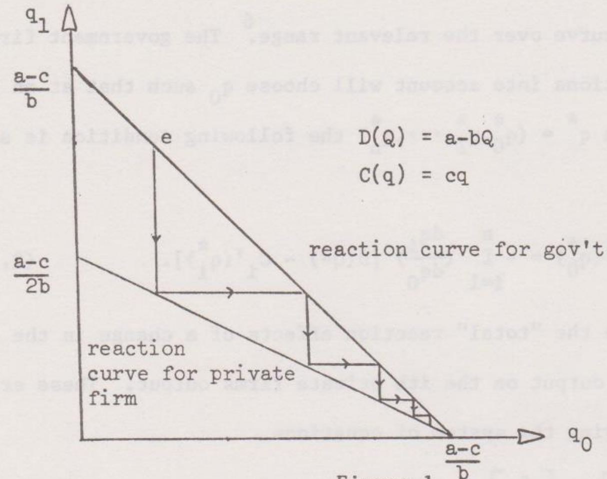


Figure 1

b) Stackleberg Mode: We turn now to the case in which the government firm takes into account the private firms' output reactions in response to a change in its own output. This is analogous to the Stackleberg leader-follower model of oligopoly with the government firm leading. From (2.2) we derive the change in a private firm's output due to a change in the output of any other firm,

$$\frac{\partial q_i}{\partial q_j} = \frac{-[q_i D'' + D']}{[2D' + q_i D'' - C_i'']}, \quad i=1,\cdots,n \quad j=0,1,\cdots,n, \tag{2.4}$$

which is negative, provided the market demand curve exhibits a diminishing marginal revenue curve over the relevant range.[6] The government firm taking these reactions into account will choose $q_0$ such that at an equilibrium allocation $q^* = (q_0^*, q_1^*, \cdots, q_n^*)$ the following condition is satisfied:

$$D(Q^*) - C_0'(q_0^*) = -\sum_{i=1}^{n} \left(\frac{dq_i}{dq_0}\right) [D(Q^*) - C_i'(q_i^*)]. \qquad (2.5)$$

Here $(dq_i/dq_0)$ are the "total" reaction effects of a change in the government firm's output on the ith private firms output. These are calculated by solving the system of equations

$$[a_{ij}] \left[\frac{d\hat{q}}{dq_0}\right] = \left[\frac{\partial \hat{q}}{\partial q_0}\right] \qquad (2.6)$$

for $[d\hat{q}/dq_0]$ where $\hat{q} = (q_1, \cdots, q_n)$ and $[a_{ij}]$ is an nxn matrix with $a_{ii} = 1$ for all i, and $a_{ij} = [\partial q_i/\partial q_j]$, $i \neq j$. If we let $m_i \equiv \frac{D(Q^*) - C_i'(q_i^*)}{D(Q^*)}$ denote the distortion between price and marginal cost for each firm, $\varepsilon_j \equiv -\frac{dq_i}{dq_0} \cdot \frac{q_0}{q_i}$ the total reaction elasticity of the ith firm to changes in the public firm's output and $s_i \equiv q_i^*/Q^*$ the equilibrium market shares, then (2.5) can be written as

$$s_0 m_0 = \sum_{i=1}^{n} \varepsilon_i s_i m_i. \qquad (2.7)$$

Thus in equilibrium the market-share weighted distortion by the government firm is equal to the market-share weighted distortion by the private firms times the reaction elasticities $\varepsilon_i$. Hence the government firm's distortion will be greater (i) the greater the market share of the other firms, (ii) the greater the distortion of the private firms, and (iii) the larger the reaction elasticities.

In the Stackleberg Mode a "first-best" allocation is not achieved as private firms are not setting their output levels such that price equals marginal cost. But it is the case that a higher value of social welfare will be achieved when the government firm uses the Stackleberg Mode rather than the Cournot-Nash mode. This follows simply because the government firm takes into account the effects its output decisions have on private firm's decisions. Condition (2.7) is a necessary condition for a second-best optimum when the government firm chooses a Stackleberg Mode of behaviour. Not surprisingly in this second-best case the government firm does not choose to price at marginal cost.

c) Reaction Function Mode:

In many cases it is reasonable to give the government firm an advantage not enjoyed by other firms - that of determining and announcing a strategy to the other firms. It is well known in the game theory literature that this may give the player announcing his strategy an advantage over other players.[7] If this is the case then all firms who could benefit from such a policy would like to do so. However, only a firm which can announce a strategy and convince all other firms that it will stick to it independently of what the other firms do will find such a strategy to be viable. The government firm is a logical candidate in this regard. Firstly, the government firm is unique in the industry not only because it acts with respect to a different set of objectives than other firms, but also because it has financial resources far greater than other firms in the industry. Thus if other firms in the industry chose to "test" its declared intention it would survive such a test even if it meant operating at a loss for some period of time.

Secondly, the private firms may view the government firm as just another extension of government policy into their environment. Consequently, they would prefer the government firm announce a set policy prior to their having to make any decisions. This would eliminate a great deal of "behavioural uncertainty" for them, or alternatively reduce the risk of unanticipated and harmful government action. If a government firm announces its strategy prior to the decision making of private firms this would give the private firms precisely the type of information they desire.

The problem of what strategy the government firm should announce will now be formulated more precisely. The government firm may choose a reaction function $\phi$ which is a function giving the government firm's output as a function of all other firms' output,

$$q_0 = \phi(q_1,\cdots,q_n).$$

An allocation $q^* = (q_0^*,\cdots,q_n^*)$ is said to be optimal if and only if

$$Q^* = \sum_{i=0}^{n} q_i^*$$

and

$$D(Q^*) = C_i'(q_i^*),\ \ i=0,1,\cdots,n.$$

Let us denote the profit of the ith firm when the government chooses a reaction function $\phi$ by

$$\pi_i(q_1,\cdots,q_n;\phi) = q_i D(Q) - C_i(q_i),\ \ i=1,\cdots,n,$$

where

$$Q = \phi(q_1,\cdots,q_n) + \sum_{i=1}^{n} q_i.$$

Once the reaction function $\phi$ has been announced to the private firms, they are faced with an oligopolistic situation with interdependencies among firms occuring through the joint effect of the market demand function and the government firm's reaction function. We shall treat this oligopoly situation as an n-person non-cooperative game. For each reaction function $\phi$ there will be a different non-cooperative game played by the private firms.

Let $\bar{q_i} = (q_1,\cdots,q_{i-1},q_{i+2},\cdots,q_n)$ i.e., the ith component deleted from the vector $\hat{q} = (q_1,\cdots,q_n)$ and when convenient let $\hat{q}$ be written as $(q_i,\overline{q}_i)$.

Suppose there exists a reaction function $\phi^*$ such that:

$$\text{(A)} \quad \pi_i(q_i^*,\overline{q}_i;\ \ \phi^*) \geq \pi_i(q_1,\cdots,q_n;\ \ \phi^*)$$

$$\text{for all } \overline{q} = (q_1,\cdots,q_n)\ \varepsilon R_+^n;$$

$$\text{(B)} \quad q_0^* = \phi^*(q_1^*,\cdots,q_n^*).$$

Property (A) requires that against the reaction function $\phi^*$, $q_i{}^*$ is the dominant strategy choice for the ith firm, i.e., the ith firm will choose $q_i{}^*$ <u>independent</u> of what other firms do. Property (B) requires that the reaction function be consistent with an optimal decision by the government firm. If such a reaction function exists we say that the reaction function $\phi^*$ <u>strongly supports</u> the allocation $q^*$.

It will now be shown that for the case of an oligopoly producing a homogeneous product such a reaction function exists. Consider the reaction function

$$q_0 = Q^* - \sum_{i=1}^{n} q_i. \tag{2.8}$$

Then, we have

$$\pi_i(q_1,\cdots,q_n;\ \ \phi) = q_i D\left[\phi(q_1,\cdots,q_n) + \sum_{i=1}^{n} q_i\right] - C_i(q_i)$$

$$= q_i D\ (Q^*) - C_i(q_i).$$

Clearly the choice $q_i{}^*$, where $D(Q^*) = C_i{}'(q_i^*)$, is optimal for the ith firm, independent of what the other firms do. Furthermore, by definition

$$q_0^* = Q^* - \sum_{i=1}^{n} q_i^*.$$

Thus the reaction function given by (2.8) satisfies both properties (A) and (B), and qualifies as a strongly supporting reaction function.

The basic idea here is quite simple. The government firm announces a strategy which effectively insures that aggregate industry output is fixed independent of what the private firms choose to do.[8] In such a situation any private firm faces a fixed price for its outputs, given by $D(Q^*)$, and the best it can do is to maximize profits by choosing its output such that marginal cost equals price. The government firm can thus attain an optimal allocation of outputs across all firms solely through the control of its own output level. Formally, the reaction function mode leads to precisely the same solution as would regulation of price by a government agency.

It is worth noting that only allocations for which marginal cost is not less than average cost for all private firms can be supported as an oligopolistic equilibrium with the appropriate reaction function. The traditional problem of increasing returns remains here as in competitive theory. It is possible however for firms to have a region of initially falling average cost. The size of this region may well provide a justification for the number of firms in the oligopoly.

In summarizing this section, we have shown that the reaction function mode dominates both the Cournot-Nash and Stackleberg modes in terms of the welfare attained. Given a "first-best" allocation, that is an allocation which a welfare maximizing planner with complete control of all output levels would choose, there exists a reaction function for the government firm such that the resulting oligopolistic equilibrium will coincide with the "first-best" allocation. In light of this result a more detailed analysis of the reaction function mode seems necessary.

## 3. Supporting Reaction Functions and Cournot-Chamberlain Oligopoly

Many oligopolistic situations are characterized by a few firms producing products which are close substitutes or complements and in making their decisions this interdependence is taken into account. Such a model is sometimes referred to as the Cournot-Chamberlain [1948] [1963] model of oligopolistic competition. In this section we consider what impact a government firm might have in such a market structure by announcing to the private firms a particular strategy in the form of a reaction function.

It is necessary to introduce some further notation at this point. Again there are n + 1 firms each producing a single good. The inverse demand function for each firm is denoted by

$$D_i(q_0, q_1, \cdots, q_n), \quad i=0,\cdots,n.$$

We now introduce the following.

<u>Assumption</u> $D_i$ is a twice continuously differentiable function defined on the interior of $R_+^{n+1}$ with the following properties:

(1) $\frac{\partial D_i}{\partial q_0}$ and $\frac{\partial^2 D_i}{\partial q_0^2}$ are uniformly bounded away from zero on $R_+^{n+1}$;

(2) $\frac{\partial^2 D_i}{\partial q_0^2} < 0$ for all $q \in \text{int } R_+^{n+1}$;

(3) All first and second partial derivatives are uniformly bounded on int $R_+^{n+1}$;

for every $i=0,1,\cdots,n$.

(1) says that a change in the output of the government firm always has an effect on the price received by all other firms, i.e., the government firm genuinely competes with all other firms in the oligopoly. The second assumption says this effect is diminishing;

(3) is a regularity condition which does not seem unnecessarily restrictive. Again all firms have cost functions $C_i(q_i)$ with the usual properties. An allocation $q^* = (q_0^*, q_1^*, \cdots, q_n^*)$ is defined to be optimal if and only if

$$C_i'(q_i^*) = D_i(q_0^*, \cdots, q_n^*), \quad i=0,1,\cdots,n.$$

That is price equals marginal cost for all firms. We shall assume that at least one optimal allocation exists, and that it is interior to $R_+^{n+1}$.

Given a reaction function $\phi$ chosen by the government firm, which gives the output of the government firm as a function of the outputs of all other firms, the n private firms face an oligopolistic situation. Each firm's profit function is given by

$$\pi_i(q_1, \cdots, q_n; \ \phi) = q_i D_i[\phi(q_1, \cdots, q_n), q_1, \cdots, q_n] - C_i(q_i)$$

$$i=1, \cdots, n, \tag{3.1}$$

and each firm's profit depends upon the output decisions of all firms. We treat this as a traditional n-person non-cooperative game with perfect information. A Cournot-Nash equilibrium of such a game is defined as an n-tuple $(q_1', \cdots, q_n')$ of outputs such that

$$\pi_i(q_i', \bar{q}_i'; \ \phi) \geq \pi_i(q_i, \bar{q}_i'; \ \phi) \quad \text{for all } q_i \geq 0,$$

$$i=1, \cdots, n.$$

A reaction function $\phi^*$ is said to <u>weakly support</u> the allocation $q^*$ if and only if

(C) $q^*$ is a Cournot-Nash equilibrium characterized by profit functions (3.1) with $\phi = \phi^*$.

(D) $q_0^* = \phi^*(q_1^*, \cdots, q_n^*)$.

We say that the reaction function weakly supports $q^*$ because it does not have the dominant strategy property which characterizes strongly supporting reaction functions. Property (C) implies that firm i will choose $q_i^*$ given the reaction function $\phi^*$ and output levels $\bar{q}_i^*$ for all other firms. Property D requires that the reaction function induces an optimal output decision by the government firm given that all other firms produce at optimal levels.

To demonstrate the existence of a weakly supporting reaction function to any optimal allocation $q^*$ we proceed again by construction. Consider the reaction function, $\phi^*$, given by

$$\phi^*(q_1, \cdots, q_n) = C + \sum_{i=1}^{n} [\beta_i q_i + \gamma_i (q_i \ln(q_i/q_i^*) - q_i)] \quad (3.2)$$

where $$\beta_i = - \frac{\partial D_i}{\partial q_i} (q_0^*, \cdots, q_n^*) / \frac{\partial D_i}{\partial q_0} (q_0^*, \cdots, q_n^*),$$

C is a constant chosen such that

$$q_0^* = C + \sum_{i=1}^{n} \beta_i q_i^*,$$

and the $\gamma_i$ are constants defined below.

The $\beta_i$ are all well defined given assumption (1) made on the demand functions. We now examine the first-order conditions to the firms' maximization problems.

$$\frac{\partial\pi_i(q_i,\bar{q}_i;\ \phi^*)}{\partial q_i} = D_i(q) + q_i\,\frac{\partial D_i(q)}{\partial q_0}\,\frac{\partial\phi^*}{\partial q_i} + q_i\,\frac{\partial D_i(q)}{\partial q_i} - C_i'(q_i),\ i=1,\cdots,n.$$

Given the reaction function (3.2) it follows that at $q^*=(q_o{}^*,\cdots,q_n{}^*)$, we have

$$\frac{\partial\pi_i(q_i^*,\bar{q}_i^*;\ \phi^*)}{\partial q_i} = 0,\ i=1,\cdots,n. \tag{3.3}$$

To establish that all firms are at a profit maximum consider

$$\frac{\partial^2\pi_i}{\partial q_i^2}(q_i,\bar{q}_i;\ \phi^*) = (2\,\frac{\partial D_i}{\partial q_i}) + (2\,\frac{\partial D_i}{\partial q_0}\cdot\frac{\partial\phi^*}{\partial q_i}) + [q_i\cdot\frac{\partial^2 D_i}{\partial q_i^2} + q_i\cdot\frac{\partial^2 D_i}{\partial q_0^2}\cdot(\frac{\partial\phi^*}{\partial q_i})^2]$$

$$+ (2q_i\cdot\frac{\partial^2 D_i}{\partial q_0\partial q_i} + q_i\cdot\frac{\partial D_i}{\partial q_0}\cdot\frac{\partial^2\phi^*}{\partial q_i^2}) - (C_i''). \tag{3.4}$$

Since $\frac{\partial\phi^*}{\partial q_i} = \beta_i + \gamma_i\,\ln(q_i/q_i^*)$ equation (3.3) can be written as the sum of the right hand terms with $\beta_i$ replacing $\frac{\partial\phi^*}{\partial q_i}$ plus the following expression

$$(\gamma_i\cdot\frac{\partial D_i}{\partial q_0}) + (\gamma_i\cdot[\frac{\partial D_i}{\partial q_0} + 2q_i\,(\beta_i\,\frac{\partial^2 D_i}{\partial q_0^2} + \frac{\partial^2 D_i}{\partial q_0\partial q_i})]\,\ln\,(q_i/q_i^*))$$

$$+ [\gamma_i{}^2\cdot\frac{\partial^2 D_i}{\partial q_0^2}\cdot q_i\cdot(\ln\,(q_i/q_i^*))^2]. \tag{3.5}$$

By assumption all first and second partial derivatives of the inverse demand functions are uniformly bounded, $\partial^2 D_i/\partial q_0{}^2$ and $\partial D_i/\partial q_0$ are uniformly bounded from zero, and $\partial^2 D_i/\partial q_0^2 < 0$. If we choose $|\gamma_i|$ large enough with sign $\gamma_i$ equal to minus sign $(\frac{\partial D_i}{\partial q_0})$ the term (3.4) can be made negative everywhere on any compact subset $\Omega$ of the interior of $R_+^{n+1}$. Thus $\pi_i(q_i,\bar{q}_i;\ \phi^*)$ is a strictly concave function in $q_i$ on $\Omega$ implying that (3.3) is both necessary and sufficient to describe the firms' optimal output choices, relative to a feasible set of joint outcomes $\Omega$.

Unless the demand functions exhibit some special separability properties it will not be possible in general to find a supporting reaction function such that the individual firm's choices have a dominant strategy property. The Cournot-Nash property of equilibrium strategies seems to be the best that can be hoped for.

In summary then, we have shown that the results of section 2 on the reaction function mode for the government firm generalize in a slightly weaker sense to the case of an oligopolistic industry producing products which are close, but not perfect, substitutes. The crucial assumption was that a change in the government firm's output level has a "significant" impact on the price any private firm gets for its product. Without this assumption, of course, the government firm could have no impact on the other firms. Given that the government firm can affect all other firms prices' its reaction function effectively 'enforces' the allocation $q^*$. Any firm that individually tries to deviate from its output choice $q_i^*$ will find that the government reacts in such a fashion as to make such a move unprofitable. As all private firms know how the government firm will react, and given that they are at the allocation $q^*$, none will individually have an incentive to change their output levels. Such an equilibrium, however, is not stable against the formation of coalitions among the private firms.

## 4. An Iterative Scheme with Incomplete Information

As we argued in the introduction, a crucial aspect of any public policy designed to influence the behaviour of private agents is the information on which such policies are based. All too often economic analysis, and in particular welfare economic analysis, proceeds on the assumption of perfect information about the economic environment. In this section we propose to examine the reaction function mode as a regulation device when the government owned firm has only limited information. In particular we deal with the problem when the government firm may not know what the cost functions of the other firms are. The analytical approach we take is the analysis of an idealized iterative scheme, with the government firm revising its policy at each iteration in response to what is observed. This approach which is common in the planning literature has its shortcomings, but it does have the merit of approximating a real world regulatory process which proceeds by trial and error. In the real world of course the number of actual iterations are very few.

The model is basically that of section 2. A single industry consisting of n+1 firms, all produce the same homogeneous good with an industry inverse demand function D(Q). Any firm in the industry, including the government firm, is assumed to know only (1) the market demand function D(Q), (2) the government firm's reaction function and (3) its own cost function. No firm has any information about other firms' cost functions. The assumption that all firms know the market demand function does not seem unreasonable, and in any case could be modified to an assumption that it was only "known" in a statistical sense without any substantive change of the results.

All variables will be indexed with a t to denote the $t^{th}$ iteration. "Time" is to be understood in this section as "planning time", and not calendar time. The calendar time between each iteration could well vary.

The iterative scheme is as follows. At this point we present it in discrete terms, although in the more formal analysis we shall use continuous time for analytical convenience.

1. Iteration t. At the $t^{th}$ stage the government firm announces to private firms a reaction function, $q_0^t = Q^t - \Sigma_{i=1}^n q_i^t$, where $Q^t$ is a parameter which is the target aggregate output in the $t^{th}$ iteration.

2. During iteration t, following the analysis of section 2(c) equilibrium occurs with private firms all producing $\bar{q}_i^t$, where $C_i'(\bar{q}_i^t) = D(Q^t)$, for $i=1,\dots,n$, and the government firm produces $\bar{q}_0^t = Q^t - \Sigma_{i=1}^n \bar{q}_i^t$. Note the government firm does not know the individual private firms' output levels or their cost functions. It does, however, know their common marginal cost which is equal to $D(Q^t)$.

3. After equilibrium has occured the government firm revises the reaction function parameter, $Q^t$, aggregate industry output, according to the following rule:

$$Q^{t+1} = \lambda[D(Q^t) - C_0'(\bar{q}_0^t)]Q^t + Q^t$$

where $\lambda$ is some positive adjustment parameter. Thus the government firm increases (decreases) the desired aggregate industry output level if it observes that its marginal cost is less than (greater than) the industry price. If its marginal cost equals industry price then no change occurs in the reaction function parameter, and the entire process stops. If not then a new reaction function is announced to the industry with the parameter $Q^{t+1}$ and the process repeats itself.

We turn now to examining what properties this process has.

Informational Privacy. At no point is information transmitted directly from the private firms to the public firm or between the private firms. Because of the dominant strategy property of the private firms' output choices they do not need to know any information about other private firms. The public firm only acquires indirect information about private firms, in that at each iteration it knows what they as a group are producing by observing $Q^t - q_0^t$.

Market Clearing. At each iteration the market clears at $Q^t$, the government firm making up the difference between private firms output and the planned aggregate output $Q^t$. This is in contrast to many regulation schemes, such as price regulation, which give rise to market disequilibrium and hence the need for rationing during the adjustment process.

Monotonicity. A desirable and important property of any iterative scheme for resource allocation is that with each iteration of the scheme the resource allocation improves in terms of some performance measure. Thus, if for practical reasons it is necessary to stop the process before it reaches a global optimum, one is assured of having moved in the right direction; starting the process will not lead to a worse allocation than the initial allocation. The iterative scheme outlined here has this feature. In continuous time the government firm's revision rule can be expressed as

$$\dot{Q} = \lambda[D(Q) - C_0'(q_0)], \qquad (4.1)$$

where $\dot{Q} = dQ/dt$.

From the private firms' optimality conditions we can write each $q_i$ as a function of Q, and using the reaction function we get

$$q_0 = Q - \Sigma_{i=1}^{n} q_i(Q) \qquad (4.2)$$

Substituting (4.1) into (4.2) gives an ordinary differential equation in the single variable Q. Differentiating the welfare function (2.1) with respect to t we get

$$\dot{W} = [D(Q) - \Sigma_{i=0}^{n} [C_i'(q_i(Q))] q_i'(Q)] \dot{Q} \qquad (4.3)$$

Substituting $C_i' \, [q_i(Q)] = D(Q)$ for $i=1,\ldots,n$ and noting that $\Sigma_{i=1}^{n} q_i'(Q) = 1 - q_0'(Q)$ (4.3) can be written as

$$\dot{W} = [D(Q) - C_0'(q_0)] \dot{Q} q_0'(Q)$$

$$= \frac{1}{\lambda} (\dot{Q})^2 q_0'(Q). \qquad (4.4)$$

As $q_0'(Q) > 0$ we have that

$$\dot{W} \begin{cases} >0 \text{ if } D(Q) \neq C_0'(q_0) \\ =0 \text{ if } D(Q) = C_0'(q_0). \end{cases}$$

Thus provided the process has not terminated, social welfare monotonically increases with each iteration.

Convergence to the Optimum. As a by-product of the analysis on monotonicity it follows immediately that the process, given a sufficient number of iterations, will converge arbitrarily close to a global optimum. The allocation which obtains in the convergent state is optimal as all firms have output levels such that price equals marginal cost.

The iterative scheme proposed thus provides a feasible means, in the absence of perfect information, by which the reaction scheme of section 2 can be implemented. As with any resource allocation mechanism, if all agents understand how the mechanism functions, it suffers from certain incentive problems. The private firms want to see aggregate output lower, and consequently price higher than does the government firm. If they understand how the government firm revises its aggregate output target then they have an incentive at each iteration to produce less than they otherwise would, i.e., operate where price exceeds marginal cost. This has the effect of having the government firm produce more and consequently raises its marginal cost. Then through the adjustment mechanism the government firm will lower industry output (or at least not increase it by as much) and raise the price.[11] How serious this incentive problem is will depend upon the degree of sophistication of the private agents, how far sighted they are and how well they understand the process the government firm is using.

5. Comparison with Alternative Policies

In this section we compare the use of a government firm in an oligopolistic industry with alternative instruments of public policy. The coverage is selective and is intended primarily as a clarification of issues for future research. In addition we confine ourselves to the market failures framework outlined in the introduction. The three main public policy alternatives to public enterprise, which are by no means mutually exclusive, are fiscal instruments, anti trust policy and direct regulation of the industry. We consider each of these in turn.

Public enterprise within an oligopolistic industry may be preferred to the traditional tax-subsidy instruments of public policy for at least two sorts of reasons. The first of these is a second-best argument. The set of feasible tax instruments is assumed to be constrained, for one reason or another, such as to preclude the attainment of certain objectives. Thus in the absence of lump-sum or unlimited profits taxation the government may decide to purchase a firm in a concentrated industry in order to generate revenue. For example, in industries extracting non-reproducible natural resources, government ownership of a firm provides a means of obtaining some of the rents going to the producers of the resource. Another argument along second-best lines has to do with risk bearing. It is often argued that the government has greater capacity for diversifying risk than do private firms and this should be taken into account in making public investment decisions.[12] Suppose some industry is identified as one where investment is particularly risky and where potentially high returns exist. Suppose also that because of this high risk, the private firms within the industry are not investing at the socially correct level. One policy to correct for this type of market failure would be to subsidize investment within this industry and tax the (random) returns appropriately. If such a policy is not feasible, then an alternative would be direct government participation in the investment process through public enterprise. The government firm because of its greater risk-bearing capacity could undertake projects which the private firms would not undertake.

A second reason public enterprise may be preferred to fiscal instruments is the inability of tax-subsidy instruments to achieve an efficient allocation of resources in a market where firms have market power. This point has recently been made by Guesnerie and Laffont [1976] who demonstrated that when a firm has sufficient market power to affect the price it receives for its product it may be <u>impossible</u> to choose a tax structure such as to achieve an efficient allocation of resources. This result indicates that a more direct means of control of the industry is necessary.

The limitations of anti-trust policy are well known and need not be repeated here. In the case of oligopoly it is impossible to legislate against strategic behaviour of a non-collusive nature. Anti-trust is best suited to dealing with collusive and non-competitive practices within an industry. Whether public enterprise has any role to play in this regard is an open question.

In light of the above discussion, it would seem that a direct form of public intervention in the industry is called for if the structural market failure is to be corrected. The form of intervention economic analysis has dealt with most commonly is direct regulation of price and/or quantities within the industry. If a regulatory agency fixes the price in the industry then the inefficiencies due to the strategic interaction of private firms is eliminated. In this paper we have shown that precisely the same outcome can be achieved through the use of a government firm in the industry. Strategic interaction among the private firms is eliminated by having the government firm behave such as to fix the price faced by all private firms. Of course the equivalence of price regulation and regulation via a government firm is only obvious in a world of complete and perfect information, with costless administration and instantaneous adjustment. If these assumptions are relaxed how do the alternatives compare?

One clear advantage of the government firm over price regulation is that it has an inherent informational advantage. The government firm knows its own technology and hence costs, and to the extent that other firms in the industry have similar costs, the government firm has partial information on these as well. The regulatory agency does not have direct information of this type and must either rely on the information provided to it by the private firms, which may be distorted, or expend resources in acquiring this information. Related to the informational argument is the question of flexibility; i.e., the ability of the policy instrument to respond to changes in cost and demand conditions. Public enterprise may dominate price regulation on these grounds if it has the informational advantage noted above, because then it could adjust its policy more quickly than a price regulator. More important perhaps, the price regulator may have limited ability to make quick responses to changes in cost or demand conditions. Before any change in price could be made it would be necessary to either pass legislation or undertake a regulatory review procedure.

Comparing the direct costs and benefits of both procedures, i.e., those exclusive of the improved resource allocation, does not give either a clear cut advantage over the other. In the case of public enterprise there is the cost of purchasing the government firm and the profits which it makes once in operation. Hopefully, as we are not concerned with decreasing cost problems, the government firm would not run at a loss over the long run. But the analysis of the iterative scheme in section 4 suggests it might well incur losses over short-run adjustment periods. The direct costs of price regulation are those incurred in running the administrative agency.

One of the major difficulties with price regulation is that, due to either imperfect information or administrative lags, the price is not set at the market clearing value. It then becomes necessary either to ration the available supply or demand, or alternatively have the government run a buffer stock scheme. The cost of either of these devices can be quite high. A government firm using the reaction function mode entirely avoids this problem. While it may face the same informational difficulties as the price regulator in estimating what the appropriate level of aggregate industry output is, it always adjusts its output such that the market clears. In this case the cost incurred is an efficiency loss as the government firm will not be producing where its marginal cost equals price; all private firms, however, will produce at levels such that price equals marginal cost.

Finally we note that both alternatives suffer from incentive problems when there is incomplete information on costs. Private firms in both cases have an incentive to behave in such a manner as to get either the price regulator or government firm to raise the price or lower the industry supply from what they would be under perfect information. It is not apparent under which of the two alternatives this incentive problem is more severe.[13]

## 6. Conclusion

This paper has considered how a government firm which competes with private firms in an oligopolistic industry can improve the allocation of resources within the industry. Three alternative behavioural rules for the government firm were considered: the Cournot-Nash Mode in which the government firm ignores the effect its output choices have on the private firms; the Stackleberg Mode in which the government firm takes these effects into account; and the Reaction Function Mode in which case the government firm announces its output strategy to the industry in the form of a reaction function. The reaction function mode was shown to dominate the other modes in terms of its ability to achieve the highest level of social welfare. Specifically, it was proved that there exists a reaction function for the government firm such that the first-best optimal allocation of outputs across firms would obtain as the resulting oligopolistic equilibrium. Furthermore we proved that this result generalized to the case of a Cournot-Chamberlain oligopoly producing differentiated products. An iterative scheme was developed by which the government firm revises its reaction function in response to observed market outcomes when it does not have any information about private firms' cost functions. The scheme was shown to have a number of desirable properties; in particular it is globally stable and converges to the global optimum.

The results of this paper suggest that on theoretical grounds public enterprise is a viable alternative to other instruments of public policy designed to influence resource allocation in oligopolistic industries. In some circumstances it appears that internal regulation via a government firm has advantages over the alternatives. The issues, however, have only begun to be explored. In order to make comparisons among alternative regulation devices it is critical to pay attention to those features, such as uncertainty, informational asymmetries or bureaucratic incentive problems which should highlight the differences. Furthermore, it is important to examine to what extent the assumptions maintained in this paper can be relaxed and yet still be able to obtain positive results. For example, the partial equilibrium assumption is clearly restrictive and extension to a general equilibrium framework would seem worthwhile. Finally an analysis of the interaction between the government firm, and other policy instruments such as tax policy and regulation is necessary. If there is more than one policy goal then it is important to coordinate the available policy instruments. The policies of public enterprise should be examined in light of this problem.

## Footnotes

1. We are not concerned with the natural monopoly argument for public enterprise. If there are regions of initially decreasing average cost for some or all of the firms in the industry these are sufficiently small relative to the size of the market to justify having more than one firm in the industry.

2. Merrill and Schneider [1966] show under somewhat restrictive assumptions that the presence of a government firm can improve market performance. However the actions available to their government firm have been implicitly restricted.

3. There is some literature on how the managers of public enterprise behave, although not specifically in the context of a public enterprise competing with private firms. For example see Lindsay [1976], McKean [1964], Parkinson [1962] and Shapiro [1973].

4. We ignore the problem of how to get the managers of the public firm to behave in the appropriate manner. This problem pervades practically all the literature on public enterprise and production of public goods. Whether in practice it would be more or less important in the case of a public firm which operates in an oligopolistic industry is an open question.

5. See Luce and Raiffa [1957], chapter 5 and chapter 7, pp. 170-173.

6. That (2.4) is negative in sign follows from the second-order conditions to the ith firms maximization problem, $2D' + q_i D'' - C_i'' < 0$, and the assumption of diminishing marginal revenue, $D'' < 0$.

7. See Luce and Raiffa [1957], pp. 91.

8. Implicit in this static model is the assumption that the government firm has sufficient productive capacity to make its reaction function credible to the private firms. In order to properly analyze capacity decisions by both the government firm and private firm an explicit dynamic analysis is called for which takes into account the strategic interaction of firms. This problem is treated in Harris and Wiens [1977].

9. The restriction that all firms outputs lie in some arbitrarily large compact subset contained in the interior of the non-negative orthant of n+1 dimensional Euclidean space does not seem unduly restrictive. Bounding output levels simply reflects the requirement that the economy has a finite amount of resources. Requiring all output levels to be strictly positive means that it always pays any firm to produce.

10. For a discussion of iterative planning procedures and their properties see Heal [1973].

11. This incentive problem is not surprising since by a theorem of Hurwicz [1972] it is known there exists no resource allocation mechanism that yields "individually rational" optima which are also "individually incentive compatible" for all agents, i.e., all agents act with respect to their true preferences. Thus in the context of the problem considered here private firms have an incentive to act as if their cost functions were different than they actually are, and this will be true for <u>any</u> resource allocation process which will yield an optimum relative to the revealed cost functions of the private firms.

12. For a statement of this argument see Arrow and Lind [1971].

13. Nor by the Hurwicz [1972] impossibility theorem mentioned in footnote 10 above is there any hope for circumventing this incentive problem without sacrificing optimality.

## References

Arrow, K.J. and Lind, R.C. [1970] "Uncertainty and the Evaluation of Public Investment Decisions", American Economic Review, 60, 364-378.

Chamberlin, E. [1948] The Theory of Monopolistic Competition, Cambridge: Harvard University Press.

Cournot, A. [1963] The Mathematical Principles of the Theory of Wealth, Homewood, Illinois: Richard D. Irwin.

Davies, D.G. [1971] "The Efficiency of Public versus Private Firms, The Case of Australia's Two Airlines", Journal of Law & Economics, 14, 149-165.

Guesnerie, R. and Laffont, J.J. [1976] "Taxing Price Makers", paper presented to the European Econometric Society Meetings, Helsinki, August, 1976.

Harris, R. and Wiens, E. [1977] "Dynamic Oligopoly, Investment in Capacity and Government Firms", Discussion Paper No. 266, Institute for Economic Research, Queen's University, Kingston.

Heal, G. [1973] The Theory of Economic Planning, Amsterdam: North Holland Press.

Hurwicz, L. [1972] "On Informationally Decentralized Systems" in Decision and Organization, ed. by R. Radner and B. McGuire, Amsterdam: North Holland Press.

Lindsay, C.M. [1976] "A Theory of Government Enterprise", Journal of Political Economy, 84, 1061-1063.

Luce, R.D. and Raiffa, H. [1957] Games and Decisions, New York: Wiley.

Martin, W.H. [1959] "Public Policy and Increased Competition in the Synthetic Ammonia Industry", Quarterly Journal of Economics, 73,

McKean, R.N. [1964] "Divergences between Individual and Total Costs within Government", American Economic Review, 54, 243-49.

Merrill, W.C. and Schneider, N. [1966] "Government Firms in Oligopoly Industries: A Short-Run Analysis", Quarterly Journal of Economics, 80, 400-412.

Niskanen, W.A. [1971] Bureaucracy and Representative Government, Chicago: Aldine.

Parkinson, C.N. [1972] Parkinson's Law and Other Studies in Administration, Boston: Houghton Mifflin.

Shapiro, D.L. [1973] "Can Public Investment Have a Positive Return?" Journal of Political Economy, 81, 401-13.

Sheahan, J. [1960] "Government Competition and the Performance of the French Automobile Industry", Journal of Industrial Economics, 7, 197-215.

12. For a statement of this argument see Arrow and Lind [1971].

13. Nor by the Hurwicz [1972] impossibility theorem mentioned in footnote 10 above is there any hope for circumventing this incentive problem without sacrificing optimality.

CARLETON ECONOMIC PAPERS 1977

77-01 The Behaviour of the Prices of Traded and Non-Traded Goods: The Canadian Case 1962-74, Michael David Bordo and Ehsan Choudhri

77-02 The Flow of Implicit Subsidies/Grants Via World Trade: Transfers between Rich and Poor, their Impact on Growth and Optimality Scale, Kanta Marwah

77-03 Airport Pricing and Investment: The Case of Toronto, Sandford F. Borins

77-04 Division of Labour in the Technology of Exchange, Elmer G. Wiens

77-05 Behaviour of a Government Firm in Oligopolistic Situations, Richard G. Harris & Elmer G. Wiens

77-06 Imperfect Capital Markets as Barriers to Education, E.G. West and M. McKee

77-07 Stabilization and Nonfederal Behaviour in an Open Federal State: An Econometric Study of the Fixed Exchange Rate, Canadian Case, Munir A. Sheikh and Stanley L. Winer

77-08 Rational Expectations and the Natural Rate of Unemployment Hypothesis, Gillian Wogin

77-09 Capitalization of Property Taxes: Vacant Properties, P.C. Gotzaman and E.G. Wiens

77-10 The Transferability of the Socio-Economic Development Models of Revolutionary Cuba, Arch R.M. Ritter

77-11 Alternative Measures of Unemployment, Douglas A. Smith

77-12 The Role of R.G. Hawtrey in Keynesian Economics and The Economics of Keynes, E.G. Davis

77-13 Government Enterprise: An Instrument for the Internal Regulation of Industry, Richard G. Harris, Elmer G. Wiens.