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Behaviour of a Government Firm in Oligopolistic Situations*

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CARLETON ECONOMIC PAPERS

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1. Introduction:

Government ownership of firms which compete either directly or indirectly with privately owned firms is not unusual in some Western countries. For example in Canada more than ten Crown corporations plus others under the control of provincial governments fall in this category.[1, 2] In this paper we will show how a government firm in an oligopolistic industry can obtain a distribution of production across firms in the industry which maximizes social welfare. The government firm can competitively constrain the pricing behaviour of the industry by announcing a suitable reaction of its level of production to the privately owned firms' levels of production.[3]

The literature on the public control of an industry has typically examined two polar cases: that of monopoly, where one firm, which is either government owned or publicly regulated, is the sole supplier to a market, and the case of a competitive industry which is regulated via tax/subsidy instruments. What has not been considered, by economists in either public finance or industrial organization, is the control of an oligopolistic industry through competition by a government firm.

Before we proceed with the analysis we should state why we believe government competition as a means of regulation is at all desirable. There are two basic reasons why such regulation may be preferred to other methods. Firstly, because oligopolistic firms can respond to changes in relative prices, taxation and subsidization of their activities may not achieve what was intended.[4] Secondly, to regulate an industry effectively it is necessary to obtain data on cost and demand conditions and to design the regulation scheme so that it can react to changes in these conditions.[5] By actually being a part of the industry a government firm can acquire a great deal of information in its day-to-day activities and it can respond accordingly when circumstances change. This can be done without going through a long and costly revision of regulations involving either the legal system or the legislative process such as is characteristic of traditional regulation schemes. Through its firm the government can exert dynamic and decentralized control on the industry.

In this paper we focus on the traditional partial equilibrium analysis of oligopoly. The basic question we ask is how a government firm should behave to maximize social welfare which is measured by consumer surplus plus producer surplus. In the course of the analysis we develop a concept which can be applied more generally - the notion of a "supporting reaction function". If the government firm can induce the private firms to achieve a certain allocation of output levels, we call such a strategy a supporting reaction function.

We proceed as follows. Section 2 considers optimal behaviour rules for a government oligopolist in three classical duopoly models. In section 3 we introduce the concept of a supporting reaction function and apply it to the case where all firms produce the same good. The case where firms produce goods which are not identical but close substitutes or complements, i.e. indirect competition, is treated in section 4. Section 5 considers the case of firm interdependence through production externalities. A discussion of our results, limitations, and further applications is given in section 6.

Throughout this paper we engage in partial equilibrium analysis with complete certainty and perfect information, ignoring "second-best", distributional, and managerial incentive problems. We assume that entry to the industry is blocked and that the government has sufficient plant capacity to make its policy credible.[6]

2. A Government Firm and Duopoly

Suppose the government owns or purchases a firm in an oligopolistic industry. What would the managers of such a firm do if they were concerned with maximizing social welfare? To keep the analysis as simple as possible we consider an industry consisting of two firms producing a homogeneous consumption good. All the results of this section apply to the general case of more than two firms. The inverse demand function for the market is given by

$$P = D(Q)$$

where

$$Q = q_0 + q_1,$$

q_0 is the output of the government firm, and q_1 is the output of the private firm. The cost functions are given by

$$C_0(q_0) \text{ and } C_1(q_1).$$

The government firm maximizes social welfare given by the conventional surplus measure,

$$W(q_0, q_1) = \int_0^{q_0+q_1} D(t)dt - C_0(q_0) - C_1(q_1).$$

The private firm maximizes profits given by

$$\pi(q_0, q_1) = q_1 D(q_0 + q_1) - C_1(q_1).$$

In the language of non-cooperative game theory W and π are the payoff functions and q_0 and q_1 the strategy variables. In this section we ignore any technical issues dealing with existence or multiple equilibria and assume first order conditions characterize the solutions to the various duopoly games we consider.

Suppose that both the government and private firm act as Cournot duopolists; that is each firm chooses its output under the assumption that the other firm will hold its output constant. The equilibrium in such a situation is the familiar Cournot-Nash equilibrium and is characterized by the conditions

$$q_1 D'(Q) + D(Q) = C_1' (q_1), \tag{2.1}$$

or marginal revenue equals marginal cost for the private firm, and

$$D(Q) = C_0'(q_1) \qquad)2.2)$$

or price equals marginal cost for the government firm. Consequently, if the government firm chooses to act as if its actions had no impact on the other firm in the industry, it will follow the first-best rule of setting price equal to marginal cost.

A special case of some interest is when both firms have identical cost functions and the cost function exhibits constant average cost. In this case the only possible equilibrium is with the government firm producing all of the market demand, i.e., $Q = q_0$, and selling it at average cost. Thus a situation starting out as an oligopoly will terminate with a government monopoly. The dynamics of the linear demand case are illustrated in Figure 1.

The arrows indicate the dynamics starting from an initial point e. It is interesting that in the identical constant cost case it is not possible for an equilibrium to occur with both firms sharing the market.

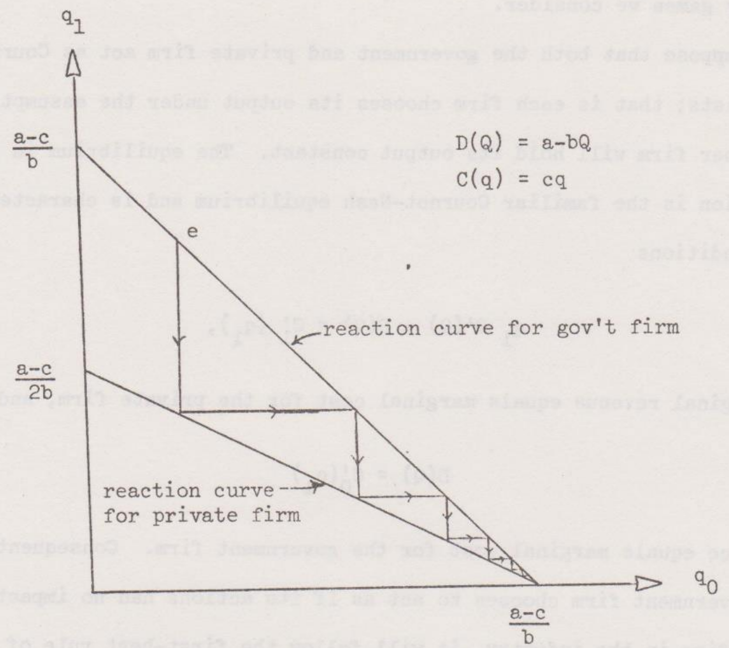

Figure 1

If the private firm acts like a Cournot-duopolist and the government firm takes this into account in making decisions then we have a situation analogous to the Stackleberg leader-follower model with the government firm leading. From (2.1) we can derive the change in the private firm's output due to a change in the government firm's output;

$$\frac{dq_1}{dq_0} = \frac{-\left[q_1 D'' + D'\right]}{\left[2D' + q_1 D'' - C_1''\right]} \qquad (2.3)$$

which is negative in sign.[7] The government firm taking this into account will choose q_0 such that the following condition is satisfied:

$$D(Q) - C_0'(q_0) = -\left(\frac{dq_1}{dq_0}\right)\left[D(Q) - C_1'(q_1)\right]. \qquad (2.4)$$

Equation (2.4) can be re-written as

$$\left(\frac{p-mc_0}{p}\right) = -\left(\frac{dq_1}{dq_0}\right)\left(\frac{p-mc_1}{p}\right), \qquad (2.5)$$

where mc_i denotes the marginal cost of the ith firm at the optimum. From (2.1) and (2.5) it follows that the government firm will never price at less than marginal cost, as $-(dq_1/dq_0)$ is positive. If we let $\frac{p-mc_i}{p} = m_i$ denote the distortion between price and marginal cost for each firm, i=0,1, $\varepsilon = -\frac{dq_1}{dq_0} \cdot \frac{q_0}{q_1}$ the elasticity of firm 1's output decreases with respect to firm 0's output increases and $s_i = q_i/Q$ the market shares. (2.5) can be written as

$$s_0 m_0 = \varepsilon \cdot s_1 m_1 \qquad (2.6)$$

Thus in equilibrium the market-share weighted distortion by the government firm is equal to the market-share weighted distortion by the private firm times the reaction elasticity ε. Hence the government distortion will be greater (i) the greater the market share of the other firm, (ii) the greater the distortion of the private firm, and (iii) the larger the reaction elasticity.

The final duopoly model we consider is the cooperative or Edgeworth duopoly model. In this case the government and private firm collude to maximize some weighted average of social welfare and private profits. There is, of course, no description of how the weights are determined in cooperative oligopoly models and the same is true in this case. An equilibrium can be described as the solution to the problem

$$\text{Max } W(q_0,q_1) \text{ subject to } \pi(q_0,q_1) \geq \bar{\pi} \tag{2.7}$$

Manipulation of the first-order conditions yields

$$\frac{(p-mc_0)}{p} = (1+\lambda)\,\frac{(p-mc_1)}{p} \tag{2.8}$$

where λ is the positive multiplier attached to the constraint in (2.7). Again we have that price is greater than marginal cost in the government firm, and the distortion is larger the more "costly" the private firm's profits are to social welfare.

This completes our discussion of the naive oligopoly models. The basic point that these models highlight is that a government firm which behaves as a traditional duopolist but with the specified objective function will never follow first-best rules unless it behaves in a Cournot fashion.

3. Supporting Reaction Function and Direct Competition

In many cases it might be reasonable to give the government firm an advantage not enjoyed by other firms - that of determining and announcing a strategy to the other firms. It is well known in the game theory literature that this may give the player announcing his strategy an advantage. If this is the case then all firms who could benefit would like to do so. However, only a firm which can announce a strategy and convince all other firms that it will stick to it will find such a strategy to be viable. The government firm is a logical candidate in this regard. In the industry the government firm is unique not only because it acts with respect to a different set of objectives than other firms, but also because it has financial resources far greater than other firms in the industry. Thus if other firms in the industry chose to "test" its declared intention it would survive such a test even if it meant operating at a loss for some periods.

The problem of what strategy the government firm should announce will now be formulated more precisely. The industry consists of n + 1 firms indexed $i=0,1,\ldots,n$ with cost functions $C_i(q_i)$ which are positive, convex, increasing and twice continuously differentiable. The industry inverse demand function is given by $D(Q)$, where $Q = \sum_{i=0}^{n} q_i$. The government firm, $i=0$, may choose a reaction function ϕ which is a function giving the government firm's output as a function of all other firms' output,

$$q_0 = \phi(q_1,\ldots,q_n).$$

An allocation $q^* = (q_0^*,\ldots,q_n^*)$ is said to be pareto optimal if and only if

$$Q^* = \sum_{i=0}^{n} q_i^* \tag{3.1}$$

$$D(Q^*) = C_i'(q_i^*),\ i=0,1,\ldots,n. \tag{3.2}$$

All firms "price at marginal cost".

Let us denote the profit of the ith firm when the government chooses a reaction function ϕ by

$$\pi_i(q_1,\ldots,q_n;\phi) = q_i D(Q) - C_i(q_i),\ i=1,\ldots,n, \tag{3.3}$$

where

$$Q = \phi(q_1,\ldots,q_n) + \sum_{i=1}^{n} q_i. \tag{3.4}$$

Once the reaction function ϕ has been announced to the private firms, they are faced with an oligopolistic situation with interdependencies among firms occuring through the joint effect of the market demand function and the government firm's reaction function. We shall treat this oligopoly situation as an n-person non-cooperative game. Notice that for each reaction function ϕ there will be a different non-cooperative game played by the private firms.

Let $\bar{q}_i = (q_1,\ldots,q_{i-1},q_{i+2},\ldots,q_n)$ i.e., the ith component deleted from the vector $q = (q_1,\ldots,q_n)$ and when convenient let q be written as $(q_i,\bar{q}_i)$. Consider the following question.

Does there exist a function ϕ^* such that:

$$\text{(A)}\ \pi_i(q_i^*,\bar{q}_i;\ \phi^*) \geq \pi_i(q_1,\ldots,q_n;\phi^*)$$

$$\text{for all } q = (q_1,\ldots,q_n)\ \varepsilon R_+^n;$$

$$\text{(B)}\ q_0^* = \phi^*\ (q_1^*,\ldots,q_n^*)?$$

Property (A) says that against the reaction function ϕ^*, q_i^* is the dominant strategy choice for the ith firm, i.e., the ith firm will choose q_i^* independent of what other firms do. Property (B) requires that the reaction function be consistent with a pareto optimal decision by the government firm. If such a reaction function exists we say that ϕ^* strongly supports the allocation q^*. We use the adjective "strongly" to distinguish it from a weaker form of supporting reaction function introduced below.

It will now be shown that for the case of an oligopoly producing a homogeneous product such a reaction function exists. Consider the reaction function

$$q_0 = Q^* - \sum_{i=1}^{n} q_i. \tag{3.5}$$

Then, we have

$$\pi_i(q_1,\ldots,q_n;\phi) = q_i D\left[\phi(q_1,\ldots,q_n) + \sum_{i=1}^{n} q_i\right] - C_i(q_i)$$

$$= q_i D\,(Q^*) - C_i(q_i).$$

Clearly the choice q_i^*; where $D(Q^*) = C_i'(q_i^*)$, is optimal for the ith firm, independent of what the other firms do. Furthermore, by definition

$$q_0^* = Q^* - \sum_{i=1}^{n} q_i^*.$$

Thus the reaction function given by (3.5) satisfies both properties (A) and (B), and qualifies as a strongly supporting reaction function. If in addition we impose the restriction that ϕ be non-negative then (3.5) can be modified to

$$q_0 = \begin{cases} Q^* - \sum_{i=1}^{n} q_i & \text{if } Q^* - \sum_{i=1}^{n} q_i \geq 0 \\ 0 & \text{if } Q^* - \sum_{i=1}^{n} q_i < 0. \end{cases} \qquad (3.6)$$

This reaction function has the reasonable property that the government firm does not produce negative output and again satisfies properties (A) and (B). Suppose $Q^* - \sum_{i=1}^{n} q_i < 0$. Then

$$\pi_i(q_1, \ldots, q_n; \phi) = q_i D\left[\sum_{i=1}^{n} q_i\right] - C_i(q_i)$$

$$< q_i\, D(Q^*) - C_i(q_i)$$

$$\leq q_i^*\, D(Q^*) - C_i(q_i^*)$$

for any vector $(q_1, \ldots, q_n)$ such that $Q^* < \sum_{i=1}^{n} q_i$. This follows as demand is downward sloping.

The basic idea of a supporting reaction function for two firms is illustrated in Figure 2. The outputs of the two firms correspond to the two axis. The heavy lines denote iso-profit contours for the private firm. Notice that the pareto-optimal allocation $q^* = (q_0^*, q_1^*)$ lies on the line $q_0 = Q^* - q_1$ which is tangent to iso-profit contour at q^*. Thus given the reaction function quoted by the government firm, the private firm will always choose to produce the pareto-optimal quantity q_1^*.

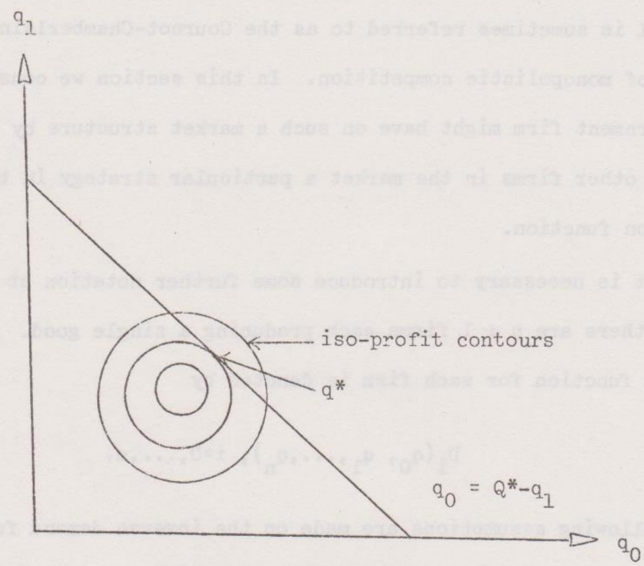

Figure 2

It is worth noting that only allocations for which marginal cost is not less than average cost for all private firms can be supported as an oligopolistic equilibrium with the appropriate reaction function. The traditional problem of increasing returns remains here as in competitive theory. It is possible however for firms to have a region of initially falling average cost. The size of this region may well provide a justification for the number of firms in the oligopoly.

4. Supporting Reaction Functions and Indirect Competition

Many oligopolistic situations are characterized by a few firms producing products which are close substitutes or complements and in making their decisions this interdependence is taken into account. Such a model is sometimes referred to as the Cournot-Chamberlain [1948] [1963] model of monopolistic competition. In this section we consider what impact a government firm might have on such a market structure by announcing to the other firms in the market a particular strategy in the form of a reaction function.

It is necessary to introduce some further notation at this point. Again there are n + 1 firms each producing a single good. The inverse demand function for each firm is denoted by

$$D_i(q_0, q_1, \ldots, q_n),\ i=0,\ldots,n. \tag{4.1}$$

The following assumptions are made on the inverse demand functions.

> D_i is a twice continuously differentiable function defined on the interior of R_+^{n+1} with the following properties:
>
> (i) $\frac{\partial D_i}{\partial q_0}$ and $\frac{\partial^2 D_i}{\partial q_0^2}$ are uniformly bounded from zero on R_+^{n+1},
>
> (ii) $\frac{\partial^2 D_i}{\partial q_0^2} < 0$ for all $q \varepsilon$ int R_+^{n+1}.
>
> (iii) All first and second partial derivatives are uniformly bounded on int R_+^{n+1};
>
> for every $i=0,1,\ldots,n$.

Assumption (i) says that a change in the output of the government firm always has an effect on the price received by all other firms, i.e. the government firm genuinely competes with all other firms in the oligopoly. The second assumption says this effect is diminishing; the last assumption is a regularity condition which does not seem unnecessarily restrictive. Again all firms have cost functions $C_i(q_i)$ with the usual properties. An allocation $q^* = (q_0^*, q_1^*, \ldots, q_n^*)$ is pareto optimal if and only if

$$C_i'(q_i^*) = D_i(q_0^*, \ldots, q_n^*), \; i=0,1,\ldots,n. \tag{4.2}$$

That is price equals marginal cost for all firms. We shall assume that at least one pareto optimal allocation exists, and that it is interior to R_+^{n+1}.

Given a reaction function ϕ chosen by the government firm, which gives the output of the government firm as a function of the outputs of all other firms, the n private firms face an oligopolistic situation. Each firm's profit function is given by

$$\pi_i(q_1,\ldots,q_n;\phi) = q_i D_i\left[\phi(q_1,\ldots,q_n),\; q_1,\ldots,q_n\right] - C_i(q_i)$$

$$i=1,\ldots,n, \tag{4.3}$$

and each firm's profit depends upon the output decisions of all firms. We treat this as a traditional n-person non-cooperative game with perfect information. A Cournot-Nash equilibrium of such a game is defined as an n-tuple $(q_1', \ldots, q_n')$ of outputs such that

$$\pi_i(q_i', \bar{q}_i'; \phi) \geq \pi_i(q_i, \bar{q}_i'; \phi) \quad \text{for all } q_i \geq 0,$$

$$i=1,\ldots,n. \tag{4.4}$$

The Cournot-Nash solution to non-cooperative games has well-known difficulties but for lack of a better equilibrium concept we shall characterize oligopolistic equilibrium in this manner.

A reaction function ϕ^* is said to weakly support the allocation q^* if and only if

(C) q^* is a Cournot-Nash equilibrium characterized by profit functions (4.3) with $\phi = \phi^*$,

(D) $q_0{}^* = \phi^* (q_1{}^*,\ldots,q_n{}^*)$.

We say that the reaction function weakly supports q^* because it does not contain the dominant strategy property which characterizes strongly supporting reaction functions. Property (C) implies that firm i will choose q_i^* given the reaction function ϕ^* and output levels $\bar{q}_i^*$ for all other firms. Property D requires that the reaction function induces a pareto optimal output decision by the government firm given that all other firms produce at pareto optimal levels.

To demonstrate the existence of a weakly supporting reaction function to any pareto optimal allocation q^* we proceed again by construction. Consider the reaction function, ϕ^*, given by

$$\phi^*(q_1,\ldots,q_n) = C + \sum_{i=1}^{n} \left[\beta_i q_i + \gamma_i (q_i \ln(q_i/q_i^*) - q_i)\right] \quad (4.5)$$

where $$\beta_i = -\frac{\partial D_i}{\partial q_i}(q_0^*,\ldots,q_n^*)/\frac{\partial D_i}{\partial q_0}(q_0^*,\ldots,q_n^*),$$

C is a constant chosen such that

$$q_0^* = C + \sum_{i=1}^{n} \beta_i q_i^*,$$

and the γ_i are constants defined below.

The β_i are all well defined given assumption (i) made on the demand functions. We now examine the first-order conditions to the firms' maximization problems.

$$\frac{\partial \pi_i(q_i, \bar{q}_i; \phi^*)}{\partial q_i} = D_i(q) + q_i \frac{\partial D_i(q)}{\partial q_0} \frac{\partial \phi^*}{\partial q_i} + q_i \frac{\partial D_i(q)}{\partial q_i} - C_i'(q_i), \ i=1,..,n. \qquad (4.6)$$

Given the reaction function (4.5) it follows that at $q^* = (q_0^*, \ldots, q_n^*)$, we have

$$\frac{\partial \pi_i(q_i^*, \bar{q}_i^*; \phi^*)}{\partial q_i} = 0, \ i=1,\ldots,n. \qquad (4.7)$$

To establish that all firms are at a profit maximum consider

$$\frac{\partial^2 \pi_i}{\partial q_i^2}(q_i, \bar{q}_i; \phi^*) = 2\frac{\partial D_i}{\partial q_i} + 2\frac{\partial D_i}{\partial q_0} \cdot \frac{\partial \phi^*}{\partial q_i} + q_i \cdot \frac{\partial^2 D_i}{\partial q_i^2} + q_i \cdot \frac{\partial^2 D_i}{\partial q_0^2} \cdot \left(\frac{\partial \phi^*}{\partial q_i}\right)^2$$

$$+ 2q_i \cdot \frac{\partial^2 D_i}{\partial q_0 \partial q_i} + q_i \cdot \frac{\partial D_i}{\partial q_0} \cdot \frac{\partial^2 \phi^*}{\partial q_i^2} - C_i''. \qquad (4.8)$$

Since $\frac{\partial \phi^*}{\partial q_i} = \beta_i + \gamma_i \ln(q_i/q_i^*)$ equation (4.8) can be written as the sum of the right hand terms with β_i replacing $\frac{\partial \phi^*}{\partial q_i}$ plus the following expression

$$\gamma_i \cdot \frac{\partial D_i}{\partial q_0} + \gamma_i \cdot \left[\frac{\partial D_i}{\partial q_0} + 2q_i \left(\beta_i \frac{\partial^2 D_i}{\partial q_i^2} + \frac{\partial^2 D_i}{\partial q_0 \partial q_i}\right)\right] \ln(q_i/q_i^*)$$

$$+ \gamma_i^2 \cdot \frac{\partial^2 D_i}{\partial q_0^2} \cdot q_i \cdot (\ln(q_i/q_i^*))^2. \qquad (4.9)$$

By assumption all first and second partial derivatives of the inverse demand functions are uniformly bounded, $\frac{\partial^2 D_i}{\partial q_0^2}$ and $\frac{\partial D_i}{\partial q_0}$ are uniformly bounded from zero, and $\frac{\partial^2 D_i}{\partial q_0^2} < 0$. If we choose $|\gamma_i|$ large enough with sign γ_i equal to minus sign $(\frac{\partial D_i}{\partial q_0})$ the term (4.8) can be made negative everywhere on any compact subset Ω of the interior of R_+^{n+1}. Thus $\pi_i(q_i, \bar{q}_i; \phi^*)$ is a strictly concave function in q_i on Ω implying that (4.7) is both necessary and sufficient to describe the firms' optimal output choices, relative to a feasible set of joint outcomes Ω.[9]

Unless the demand functions exhibit some special separability properties it will not be possible in general to find a supporting reaction function such that the individual firm's choices have a dominant strategy property. The Cournot-Nash property of equilibrium strategies seems to be the best that can be hoped for. If one complicated the analysis of oligopoly by introducing specific conjectural variations on the part of the private firms this could presumably be handled in a manner similar to that used here. This line of investigation does not seem worth pursuing however, given the arbitrary nature of the conjectural variations chosen in traditional oligopoly models.

5. Externalities: A Reaction Function Approach:

The concept of regulating market activity via the announcement of a reaction function by a government owned firm can be applied in any situation where there exist significant interdependencies between firms which are recognized by all concerned. A classic example of this type of situation is that of production externalities amongst a small number of firms. Typically economic theorists have concentrated upon tax-subsidy type solutions to externality solutions or alternatively following the literature stemming from the Coase theorem, co-operative bargaining as a solution. In many cases, however, a non-cooperative approach to the problem may be more realistic and there is no reason to expect an efficient allocation of resources to result in these circumstances. In order to illustrate the applicability of the concept of a supporting reaction function we consider a particular type of externality problem: that of input interdependencies in a single industry.

Suppose there are n+1 firms in the industry, all producing the same homogeneous good where each firm has a production function

$$f^i(x_i,X),\ i=0,1,\dots,n,$$

which gives the output of the good produced by the ith firm as a function of the input employed by the ith firm and the aggregate amount of input employed in the industry

$$X = \sum_{i=0}^{n+1} x_i$$

If all firms are competitive in the product market, and the real factor price of the input is w, then provided each firm behaves in a Cournot fashion the individual optimality conditions are given by

$$f^i_{x_i} + f^i_X = w, \; i=0, 1, \ldots, n; \qquad (5.1)$$

i.e., the private marginal product of the factor will be set equal to its price. The social optimality conditions on the other hand are given by

$$f^i_{x_i} + \sum_{j=0}^{n} f^j_X = w, \; i=0, 1, \ldots, n. \qquad (5.2)$$

The market failure arises from the familiar divergence of private and social marginal products.

Suppose that the 0th firm is purchased by the government and it announces its input decision policies in the form of a reaction function,

$$x_0 = \phi\,(x_1, \ldots, x_n)$$

Individual optimality conditions for the private firms now become

$$f^i_{x_i} + f^i_X\,(1+\phi_i) = w \qquad (5.3)$$

If ϕ is given by

$$\phi(x_i, \ldots, x_m) = c + \sum_{i=1}^{n} \beta_i x_i, \qquad (5.4)$$

where

$$\beta_k = \left[\sum_{i=0}^{n} f^i_{x_i}\,(x^*_i, X^*) \,/\, f^k_{x_k}\,(x^*_k, X^*)\right] -1, \qquad (5.5)$$

$$c = x^*_o - \sum_{i=1}^{m} \beta_i x^*_i,$$

and $x = (x^*_0, \ldots, x^*_n)$ is the pareto-optimal allocation of inputs, then substitution of (5.5) and (5.4) into (5.3) will yield the pareto-optimal conditions. The term β_k can be expressed as the ratio,

$$\frac{SMP - PMP_k}{PMP_k},$$

i.e. the ratio of the difference between private and social marginal products of the kth firm's input to the private marginal product. The reaction function given by (5.4) and (5.5) weakly supports the pareto optimal allocation of inputs. Note that higher order terms may be added to (5.4) to ensure the uniqueness of the Cournet-Nash equilibrium.

In some cases we can get a strongly supporting reaction function. Assume that the production function can be written

$$f^i (x_i, X) = h_i(x_i) + aX.$$

The individual and social optimality conditions can be expressed respectively as:

$$h_i'(x_i) + a = w$$

$$h_i'(x_i) + a\ (n+1) = w.$$

Suppose the government firm now chooses a reaction function given by

$$x_0 = c + n \sum_{i=1}^{n} x_i \qquad (5.6)$$

where the constant c is chosen such that

$$c + (n+1) \sum_{i=1}^{n} x_i^* = \sum_{i=0}^{n} x_i^*.$$

The private production function now becomes

$$h_i(x_i) + a\left[c + n\sum_{k=1}^{n} x_k + \sum_{k=1}^{n} x_k\right]$$

$$= h_i(x_i) + a(n+1)\sum_{k=1}^{n} x_k + ac.$$

In this case the individual firm's dominant strategy is to choose $x_i = x_i^*$, and consequently the reaction function given by (5.6) strongly supports the pareto-optimal allocation of inputs. In this case the government firm induces the private firms to choose the correct input levels by promising to match each additional unit of input purchased by the private firm with n units of input purchased by the government firm.

6. Conclusion

Economists have long been concerned with the design of mechanisms by which the public might exercise some control over private industry. In this paper we have examined how public ownership of a single firm in an oligopolistic industry could be used to achieve some control over the industry as a whole. The central concept introduced was that of a supporting reaction function; that is the government firm announces to the industry its strategy in the form of a reaction function such that the other firms competing in an environment characterized by this reaction function, demand and cost conditions will achieve an equilibrium which is socially optimal.

There are a number of directions in which this research could be extended. The analysis here was partial equilibrium and extending it to a general equilibrium setting would seem worthwhile. Secondly, a major problem with any regulation scheme is acquiring the necessary information on cost and demand conditions. In this paper we have used the traditional perfect information assumption of classical oligopoly theory. Consideration of issues of imperfect information and the design of incentive schemes to acquire information are as important in reaction function regulation schemes as with any other regulation mechanism. Finally, a scheme is required to ensure that the managers of the government firm faithfully execute government policy.

FOOTNOTES

1. Some of the major Crown corporations listed in the Financial Administration Act, Government of Canada, are Canadian National Railways, Canadian Broadcasting Corporation, Air Canada, Central Mortgage and Housing, Atomic Energy of Canada, Polymer, Eldorado Nuclear and Petro Canada. Crown corporations of provincial governments include Ontario Hydro, B.C. Hydro and Quebec Hydro. The federal and provincial governments also take equity positions in other corporations. For example Pacific Western Airlines is owned by the Government of Alberta; De Havilland Aircraft of Canada is owned by the Government of Canada.

2. For case studies see Sheahan [1960] on the French automobile industry and Davies [1971] on Australia's airlines. For further references see Merrill and Schneider [1966], footnote 1.

3. With the exception of a paper by Merrill and Schneider [1966] we are not aware of any explicit analysis of this problem. They show under somewhat restrictive assumptions that the presence of a government firm can improve market performance. However the behaviour of their government firm has been implicitly restricted. See also Kahn [1970] for a general discussion on the public regulation of industry.

4. Guesnerie and Laffont [1976].

5. Weitzman [1974] [1977].

6. Harris and Wiens [1977] consider the case where firms can increase capacity through investment.

7. That (2.3) is negative in sign follows from the second-order condition to firm 1's maximization problem,

$$q_1 D'' + 2D' - C_1'' < 0,$$

provided we assume in addition that the market demand curve has a diminishing marginal revenue curve, D" negative, over the relevant range.

8. We were unable to show that the equilibrium against ϕ^* is unique. Thus the oligopoly game might settle at an equilibrium point other than the desired one if the initial point is not the equilibrium point.

9. The restriction that all firms outputs lie in some compact subset contained in the interior of R_+^{n+1} does not seem unreasonable. Bounding output levels simply reflects the requirement that the economy has a finite amount of resources. Requiring all output levels to be strictly positive simply means that it always pays any firm to produce.

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