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Investment in Capacity and a Normative Theory of the Dominant Public Firm*

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by

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1. Introduction

The theory of public enterprise has been almost exclusively concerned with the case of a single firm which supplies all demand forthcoming in that market - the classic natural monopoly. Yet it is increasingly common to observe public firms which compete with, or cooperate with, private firms in the same industry, and often supply a significant share of market output in what appears as an oligopolistic market structure.[1] Economic analysis has had surprisingly little to say about such firms, perhaps because they do not fall readily into the market failure rationale for public intervention.[2] An early exception to this was a paper by Merrill and Schneider (1966). In a series of papers (Harris and Wiens (1979), Harris (1978), Harris (1979), Wiens (1978a) (1978b)) we have developed a normative theory for a public firm which is dominant in its industry. The analysis in these papers is entirely static, and concentrates on price, output, and entry decisions of the public firm given that its objective is to maximize some suitably defined measure of social welfare. The static limitation in this analysis, however, is quite severe given the nature of the suggested rules the public firm might follow.

A dominant public firm is one which is capable of declaring a particular policy, say on price or output, and then having other (private) firms react to this policy. In light of this capability it chooses a policy to maximize social welfare. In Harris and Wiens (1979) we considered the policy of announcing a reaction function which states, in effect, that the public firm is willing to meet any demand forthcoming at a certain price, the price at which the industry marginal cost curve and market demand curve intersect. If the private firms believed this reaction function to be credible they would take price parametrically, produce where price equals marginal cost, and a first-best solution would be achieved. However, the difficulty lies in the credibility of the reaction function announced by the government firm. If one admits a short-long run distinction to the analysis, private firms could, by cutting back their output below the short-run social optimum, force the government firm to run at capacity output and thus raise price. Indeed it might well be profitable for the private firms to do so. In a static timeless framework, capacity limitations are not very meaningful.

The purpose of this paper is to undertake an explicit dynamic analysis of this problem; in particular to examine how a dominant public firm might structure its decisions to invest in capacity and the consequent price and output policies of all firms in the industry. In order to get a (hopefully) better understanding of the problem and the proposed solution the paper proceeds in order of increasing complexity. Section 2 considers a simple two-period, two-firm model with capacity decisions being made in the first period. Section 3 goes on to consider an infinite horizon model which allows for a precise distinction between the short and long-run. Finally section 4 deals with the situation in which there are multiple private firms, and use is made of differential game theory in order to characterize equilibrium paths. A conclusion provides some interpretation and qualifications to the results obtained.

2. A Two-Period Model

Consider an industry consisting of two firms, one public and one private, both of which produce a homogeneous good to supply a single market.[3] Production and sales occur in the second period while capacity decisions occur in the first period; output in the second period is constrained by the capacity decisions in the first period. The public firm is dominant and can state its intended output and capacity policy in the first period, which it must then abide by. The private firm chooses its own capacity and output in light of this policy.[4]

Some notation: public and private variables are indexed with a subscript i=0,1 respectively; output is denoted by q_i, capacity measured in units of output by k_i, cost functions by $c_i(q_i,k_i)$, and a market (inverse) demand curve by $D(Q)$, $Q = q_0 + q_1$. The private firm's objective is to maximize its profits, $q_1D(Q) - c(q_1,k_1)$, while the public firm seeks to maximize social welfare which we take to be measured by the welfare function

$$W = u(Q) - c_0(q_0,k_0) - c_1(q_1,k_1) \tag{2.1}$$

where $D(Q) \equiv u'(Q)$; thus W is a conventional surplus measure.

Suppose the public firm were to announce an output policy for the second period of the form proposed in Harris and Wiens (1979). Specifically

$$q_0(k_0) = \begin{cases} Q^*-q_1 & \text{if } k_0 \geq Q^* - q_1 \geq 0 \\ k_0 & \text{if } Q^* - q_1 > k_0 \\ 0 & \text{if } Q^* - q_1 < 0, \end{cases} \tag{2.2}$$

where Q^* is the solution to max W subject to capacity constraints. Basically (2.2) says that provided capacity constraints do not bind the public firm will produce such as to ensure target industry output is met. However, if the private firm cuts back its output sufficiently, then the public firm will produce at capacity output. Given (2.2) the state of affairs facing a private firm is depicted in figure 1. The demand curve facing the private firm given the policy

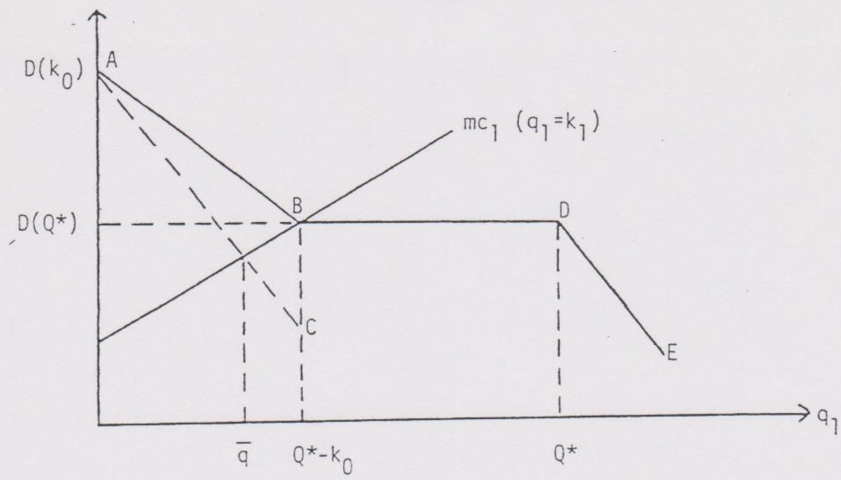

Figure 1

in (2.2) is the line ABDE. Note that the demand curve becomes perfectly elastic at the socially optimal price $D(Q^*)$, over the range BD. If $k_0<Q^*$, then there will always exist a downward sloping portion AB, over which the government firm is producing at capacity output. If $k_0 \geq Q^*$, then no such portion will exist and the government firm can always ensure at least Q^* is produced. If $k_0=k_0^*$, the optimal k_0 solution to (2.2), then the marginal cost curve of the private firm, mc_1, holding $q_1=k_1$, will intersect the demand curve at B. The marginal revenue curve is given by AC with a "jump" at C to become BD. It is clear that the marginal cost curve intersects the marginal revenue curve at two output levels, $\bar{q}$ and Q^*-k_0. The socially desirable output level for the private firm is $q_1=Q^*-k_0$. But it is quite possible that the profit maximizing choice for the private firm is to produce at $\bar{q}$; this will certainly be the case for example if mc_1 is perfectly elastic. It seems then that given the capacity constraint on the public firm and the reaction function (2.2) it may be desirable for the

private firm to install less capacity than is desirable and produce at a lower output causing industry price to rise above marginal cost.

Is there any way to avoid this dilemna? It does not appear so. The public firm can force the private firm to produce where price equals marginal cost (including incremental capacity cost) by installing capacity $k_0=Q^*$. The demand curve for firm 1 then became elastic over the entire range $(0,Q^*)$ and the private firm will produce $q_1=k_1=q_1^*$. The problem, of course, is that now the public firm will be carrying excess capacity, $Q^*-q_0^*$, and hence operating inefficiently. Another alternative which suggests itself is for the public firm to announce not only an output policy, but a capacity policy in the first period, threatening to "punish" socially incorrect capacity decisions by the private firm. For example, suppose the public firm announces in addition to (2.2) a capacity policy in period 1 given by

$$k_0 = \begin{cases} Q^*-k_1 & \text{if } \geq 0 \\ \\ 0 & \text{otherwise.} \end{cases} \tag{2.3}$$

Note we assume that in period 1 both firms are allowed to make moves and countermoves until their capacity decisions are in equilibrium against each other. The private firm, given a capacity choice k_1 in the first period, faces a demand curve as in figure 2. Since q_1 must not exceed k_1, the private firm will choose $q_1' \leq k_1$ such as to maximize profits. Thus, depending on the position of the marginal cost curve of output, the firm will produce either at k_1, or possibly with $mr_1=smc_1$; i.e. it will solve

$$\pi(k_1) \equiv \max_{q_1 \leq k_1} q_1 D(Q^*-k_1-q_1) - c_1(q_1,k_1). \tag{2.4}$$

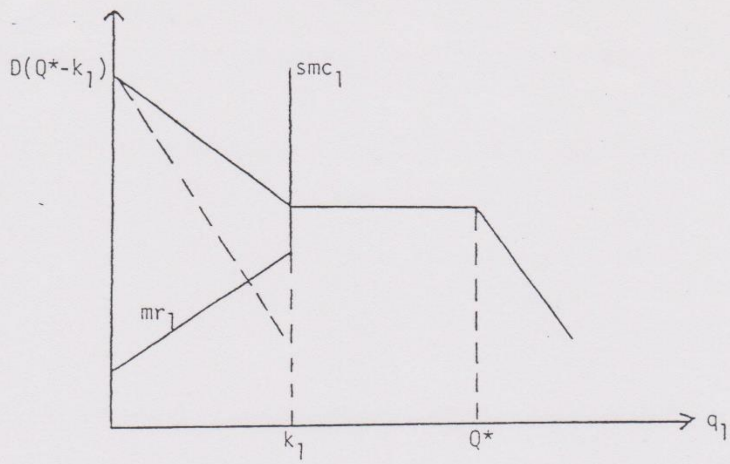

Figure 2

Now

$$\pi_1'(k_1) = \begin{cases} -q_1'D'(Q^*-k_1+q_1') - c_{1k}(q_1',k_1) & \text{if } q_1'<k_1 \\ D(Q^*) - c_{1q}(q_1',k_1) - c_{1k}(q_1',k_1) & \text{if } q_1'=k_1. \end{cases}$$

By construction, if $k_1=q_1'=k_1^*$, then $\pi'(k_1^*) = 0$. But of course if $k_1=k_1^*$, it is quite possible that in (2.4) the solution is $q_1'<k_1^*$. In general it is possible that $\pi_1'(\hat{k}_1) = 0$, with $q_1'<\hat{k}_1$; i.e., private firms operate with excess capacity and the chosen capacity level $\hat{k}_1$, may be greater or less than the socially desirable capacity level, k_1^*.

What is happening is that the essential irreversibility of the capacity decision on the part of both firms in the first period makes any sort of declared "punishment" by the public firm subject to manipulation by the private firm; there appears no way out of this problem. What is missing in the present treatment is the scope for further moves and countermoves in capacity decisions which may depend upon the outcome of decisions in previous periods. An "open-ended" model which allows for this realistic possibility is presented in the next section.

3. Dynamic Reactions and Long-Run Analysis

It seems clear from the last section that when some irreversibilities are present, including notably the fixed nature of capacity in the short-run, there is no way a dominant public firm can announce policies such as to ensure full optimality is attained. The question remains, however, whether it is possible for some sort of long-run optimum to be attained, perhaps at the expense of short-run optimality. Closely related to this is how the public firm structures its policies in light of observed market outcomes over time. In order to treat these problems we move to a continuous-time infinite horizon framework; the former for analytical convenience, while the latter yields potential asymptotic states as suitable idealizations of a long-run in which any short-run irreversibility can be removed.

The analysis is much as before, except costs include an operating cost, cq_i, and a capacity installation and maintenance cost function $G_i(I_i)$, where capacity K_i, again measured in output units, evolves according to the equation $\dot{K}_i = I_i - \delta K_i$ where $\delta > 0$ is an exponential rate of depreciation.[5] $G_i(\cdot)$ is assumed to be increasing in $|I_i|$ and strictly convex. Again the public and private firms are indexed i=0,1 respectively. Private profit and welfare are defined respectively as

$$\pi_1 = \int_0^\infty e^{-rt}[q_1 D(Q) - cq_1 - G_1(I_1)]dt \qquad (3.1)$$

$$W = \int_0^\infty e^{-rt}[u(Q)-c(q_0+q_1)-G_0(I_0)-G_1(I_1)]dt, \qquad (3.2)$$

with r the common social and private rate of time discount. Initial conditions $K_0(0)$ and $K_1(0)$ are given. The demand function D(Q) is assumed for the moment to be time independent. The socially optimal solution is obtained by maximizing (3.2). We shall assume the socially optimal trajectories converge to finite and non-zero steady-state values (K_0^*, K_1^*) and $q_0=K_0^*$ and $q_1=K_1^*$, i.e., no excess capacity is present in the long-run optimum. For convenience we note that the (K_0^*, K_1^*) satisfy the conditions

$$G_i'(\delta K_i^*)(r+\delta) = D(K_0^*+K_1^*) - c, \quad i=0,1. \qquad (3.3)$$

A dominant public firm controls only (q_0, I_0) at any time, and thus, similar to the last section, at any instant his capacity is fixed and thus his scope for punishing restrictive output policies of the private firm is limited. Over time, however, the public firm can expand or contract its capacity and can do so in response to the observed policies of the private firm. One particularly simple policy the public firm might announce is the following.

$$\begin{aligned} q_0(t) &= K_0(t) \\ I_0(t) &= \delta K_0(t)+r(K_1^T-q_1(t))+F(Q^T-K_0(t)-q_1(t)). \end{aligned} \qquad (3.4)$$

(3.4) says that the public firm will always operate at capacity, and its net additions to new productive capacity are proportional to the discrepancy between the long-run target capacity of the private firm, K_1^T, and the current output of the private firm plus a term reflecting discrepancies from long-run target <u>industry</u> output. The speed of adjustment in the "excess capacity" term is precisely proportional to the discount rate, r. With such a rule if the private firm is producing below its long run capacity the public firm will expand

and over time ceteris paribus the industry price will fall. The function $F(x)$ is a differentiable sign-preserving function of x, with $F'(x) > 0$. It's role in the analysis will become clear shortly.

What would be the optimal policy for the private firm faced with (3.4)? Since the evolution of $K_0(t)$ is determined completely by the private firm's actions this can be treated as a conventional maximization problem. The current value Hamiltonian for the private firm is

$$H = q_1 D(K_0+q_1)-cq_1-G_1(I_1)+\lambda_0\{r(K_1^T-q_1)+F(Q^T-K_0-q_1)\}$$
$$+\lambda_1(I_1-\delta_1 K_1). \qquad (3.5)$$

Maximizing H with respect to q_1, subject to the capacity constraint, $q_1 \leq K_1$, yields two possibilities; either $\hat{q}_1<K_1$, or $\hat{q}_1=K_1$. The first-order condition at the optimum $\hat{q}_1$ is

$$\hat{q}_1 D'(\hat{q}_1+K_0)+D(\hat{q}_1+K_0)-(c+\lambda_0 r+\lambda_0 F') = \begin{cases} 0 & \text{if } \hat{q}_1<K_1 \\ \geq 0 & \text{if } \hat{q}_1=K_1. \end{cases}$$

The smaller (in absolute value) λ_0, the more likely it is the capacity constraint will not bind. The reason is simply that since λ_0 is in general negative, a small λ_0 in absolute value terms means the shadow cost to the private firm of increasing the public firm's capacity is low. Thus the benefits to be had from operating with output below capacity are greater. For convenience let $\hat{q}_1=h(\lambda_0,K_0)$ denote the optimal output solution for the private firm at any t, and $\tilde{q}_1$ the unconstrained solution to max $q_1 D(K_0+q_1)-cq_1-\lambda_0 rq_1$. Clearly $\hat{q}_1=\min(K_1,\tilde{q}_1)$.

The dynamics of this problem are described in the equations:

$$\dot{K}_0 = r[K_1^T-\hat{q}_1]+F(Q^T-K_0-\hat{q}_1),$$
$$\dot{K}_1 = I_1(\lambda_1)-\delta K_1 \qquad (3.6)$$

Let

$$A_0 = r\lambda_0-\hat{q}_1 D'(K_0+\hat{q}_1)-[\hat{q}_1 D'(K_0+\hat{q}_1)+D(K_0+\hat{q}_1)-c-\lambda_0 r-\lambda_0 F']\frac{\partial\tilde{q}_1}{\partial K_0}$$

$$B_0 = r\lambda_0-\hat{q}_1 D'(K_0+\hat{q}_1)-\lambda_0 F'(Q^T-K_0-\hat{q}_1)$$

Then

$$\dot{\lambda}_0 \varepsilon \begin{cases} A_0 & \text{if } \tilde{q}_1<K_1 \\ [A_0,B_0] & \text{if } \tilde{q}_1=K_1 \\ B & \text{if } \tilde{q}_1>K_1. \end{cases} \qquad (3.7)$$

Similarly define

$$A_1 = (r+\delta)\lambda_1,\ B_1 = (r+\delta)\lambda_1-\lambda_0 F'(Q^T-K_0-\hat{q}_1)-[D(K_0+\hat{q}_1)-c-\lambda_0 r]-\hat{q}_1 D'(K_0+\hat{q}_1)$$

Then

$$\dot{\lambda}_1 \varepsilon \begin{cases} A_1 & \text{if } \tilde{q}_1\leq K_1 \\ B_1 & \text{if } \tilde{q}_1>K_1. \end{cases} \qquad (3.8)$$

Notice that we have multi-valued differential equations for the co-state variables when $\tilde{q}_1=K_1$, as the Hamiltonian is not differentiable at that point in K_0.[6] The dynamics of the shadow prices depend upon whether the private firm operates with or without excess capacity.

The problem arises as to whether or not the above system converges to a steady-state, given optimally chosen initial (λ_0,λ_1). If the system does not converge to an interior steady-state there are two possibilities; either $K_0 \to +\infty$ or $K_0 \to 0$.[7] In the former case industry price is driven to zero and we can assume that it is not optimal for the private firm to choose such a trajectory. If K_0 leads to zero necessarily $\dot{K}_0 = r(K_1^*-q_1)+F(Q^T-K_1) \to 0$. Now only states in which the private firm does not operate with excess capacity are admissable long-run equilibrium. In fact we must have $\tilde{q}_1>\overline{K}_1$, where $\overline{K}_1$ denotes the steady-state value of K_1. This follows as (3.8) implies a steady-state value of $\lambda_1=0$ if $\tilde{q}_1\leq\overline{K}_1$, which is inconsistent with a positive $\overline{K}_1$, as $G'(\delta\overline{K}_1)=\lambda_1$. Therefore if $\dot{K}_0=0$, and $K_0=0$,

$$r(K_1^T-\overline{K}_1) = -F(Q^T-\overline{K}_1). \tag{3.9}$$

The unique solution, $\overline{K}_1$, to (3.9) depends positively on Q^T, the target output parameter in the government firm's reaction function. By choosing Q^T large, $\overline{K}_1$ can be made arbitrarily large. Consequently, it is always possible to choose a sufficiently large Q^T, and hence $\overline{K}_1$, that industry price could be driven below "long-run" average cost and the steady state with $(K_0,K_1)=(0,\overline{K}_1)$ would not qualify a long-run optimum for the private firm. The only possibility which remains then is that the long-run dynamics converge to an interior steady-state in which both firms produce positive output. What can be said of this situation?

Similar reasoning to that in the previous paragraph shows that in the long-run equilibrium the private firm must be capacity constrained. Thus using $\dot{\lambda}_0=B_0=0$ and $\dot{\lambda}_1=B_1=0$ and combining yields the following equation:

$$(r+\delta)\lambda_1-(D-c)+\lambda_0 r-\lambda_0 F'-\hat{q}_1 D' = 0$$

or

$$(r+\delta)G'(\delta K_1) = D(Q)-c. \tag{3.10}$$

This together with the $\dot{K}_0=0$ equation,

$$(r(K_1^T-K_1)+F(Q^T-Q) = 0 \tag{3.11}$$

yields two equations in two-unknowns (Q,K), whose solution yields the steady-state values (Q', K_1'). The role of the $F(\cdot)$ function now is clear. One has to choose an $F(\cdot)$ with the desired properties such that the unique solution to (3.10) and (3.11) is (Q^*, K_1^*). With such an $F(\cdot)$ it follows that the long-run optimal solution for the private firm faced with the public firm's policy as described by (3.4) is to produce at the optimal level of output q_1^*, with no excess capacity. Secondly, as Q^* is the resulting long-run industry aggregate output, and $Q^*=K_0+K_1^*$, it follows that $K_0=K_0^*$, and hence the public firm is also operating at its long-run desired output. It is important to emphasize that only long-run optimality is achieved in the sense that the steady-state values of the two programmes co-incide. The dynamic paths of $K_0(t)$ and $K_1(t)$ induced by the private firm will differ from the socially optimal dynamics.

An interesting feature of this scheme is that the "targets" set, (Q^T, K_1^T), in the public firm's policy function (3.4) do not necessarily co-incide with the long-run desired output and capacity, (Q^*, K_1^*); in general one target will be under attained and one over attained in the steady-state. It is important that (Q^T, K^T) be allowed to differ from (Q^*, K_1^*) only in that by choosing these appropriately, you can prevent the private firm from choosing a path which takes the public firm out of the market. Thus for example a high Q^T, makes it quite costly for the private firm to "drive" out the public firm.

In order to get a better feel for what is going on we consider a particular example. There are two identical firms with quadratic adjustment costs, $\gamma_0 I + \frac{1}{2}\gamma_1 I^2$, and a linear inverse demand function $P=\alpha-\beta Q$. The steady-state social optimum is given by

$$K_1^* = K_2^* = [(\alpha-c)-(r+\delta)\gamma_0]/[(r+\delta)\gamma_1\delta+2\beta] \qquad Q^*=2K_1^* .$$

We take $F(Q^T-K_1-q_1)$ to be a linear function $a(Q^T-K_1-q_1)$, $a>0$. In this case (3.10) and (3.11) become

$$(r+\delta)(\gamma_0+\gamma_1\delta K_1) = \alpha-\beta Q-c \tag{3.12}$$

$$r(K_1^T-K_1)+a(Q^T-Q) = 0 \tag{3.13}$$

Graphing these in figure 3 yields the two curves as shown. By choosing a,

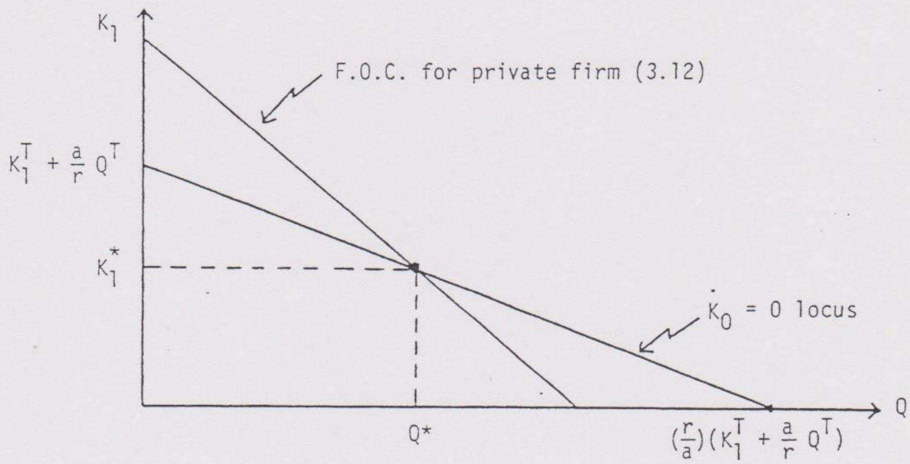

Figure 3

K_1^T and Q^T appropriately one can always ensure that the loci intersect at (Q^*,K_1^*).

The dynamics for this problem are quite complicated as transition between phases with and without excess capacity change the relevant dynamics. However some idea can be gotten of what is going on in certain cases. Suppose we take the symmetric example above with both firms starting with the same capacity $K_1(0)=K_0(0)$, with the initial capacity below the long-run optimal value. In figure 4 the social optimal trajectory is AA*, in the (K_0,K_1) plane.

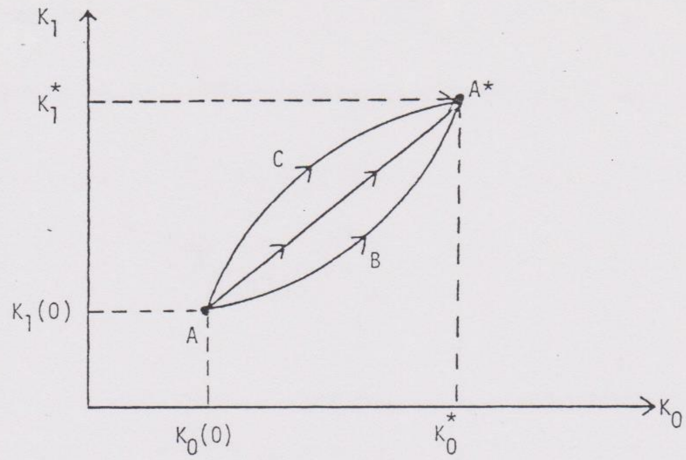

Figure 4

Assume that along the trajectory chosen by the private firm it does carry excess capacity. The public firm's capacity then evolves as $\dot{K}_0 = r(K_1^T - K_1) + a(Q^T - K_1 - K_0)$. There are two forces at work on the private firm's investment decision relative to the social optimum. First, the market power of the private firm, since it is a price setter, at least intertemporally, induces it to slow down investment in capacity in order to keep price higher, longer. On the other hand, slowing down its rate of capacity growth brings retaliation by the public firm in the form of increased capacity expansion and hence lower prices in the future. Depending upon the parameters of the reaction function one or the other will dominate. In figure 4 the path ABA* is one in which K_1 grows slower than K_0 (relative to AA*) and hence the desire to keep current prices high dominates. Along the path ACA*, the retaliatory effect of the public firm induces the private firm to move to the long-run optimum at a rate faster than the social optimum. Both types of paths have their short-run costs;

there will be both dynamic productive inefficiency in that along any resulting output path the minimum cost aggregate investment policy will not be followed, and in addition the consumer price at each instant will differ from each firms' appropriate dynamic marginal cost.

What then can be said of the short-run costs vs. long-run benefits of such a scheme? If the discount rate is very high and both firms are a long way from the steady-state, then the short-run dynamics will dominate and there seems to be little that can be done by a public firm. Some other more direct and immediate form of intervention is called for. On the other hand if the discount rate is low, or the firms are not too far from the steady-state, then long-run considerations may dominate short-run dynamics and the dominant public firm could well "regulate" the industry in the manner suggested.

The assumption of stationary demand is clearly restrictive. Dynamic considerations take on additional importance when an industry is in an expanding phase and growth in industry capacity is called for. To amend the analysis properly one needs to explicitly account for both growth in demand and dynamic entry considerations. The latter is particularly difficult and existing theory has not yet resolved how to incorporate dynamic entry properly into an oligopoly model.[8] There are two cases of non-stationary demand which can be easily incorporated into the existing analysis leaving the entry question aside. One is simply to let the demand function depend on time, $D(Q,t)$, but assume that as t gets large, growth slows and eventually demand approaches a stationary value $D^*(Q) = \lim_{t \to \infty} D(Q,t)$. The previous analysis goes through in a straightforward manner. The difficulty of course is that the long-run may be the "very" long-run. Slightly more satisfactory than this is to use a "demand growth" adjusted scheme. Suppose per capital demand is $d(q)$ and the market population is growing at an exponential rate n. With exponential growth of course the model of the firm used so far is inappropriate and we need a model where long-run firm size is indeterminate. Consequently let the adjustment cost function for each firm be the same function $H(I,K)$, where $H(\cdot)$ is an increasing convex, and linear homogeneous function of (I,K). Defining $v=I/K$, we have $H(I,K) = Kh(v)$, with $h', h''>0$. Let $z_i=q_i/e^{nt}$ and $k_i=K_ie^{nt}$, and hence $\dot{k}_i=k_i(v_i-\delta-n)$. With $Q=q_0+q_1$, the aggregate utility function is

$$u(Q,t) = \int_0^{(z_0+z_1)e^{nt}} d(\tau/e^{nt})d\tau \equiv u(z_0+z_1,t).$$

Thus the welfare problem becomes

$$\max \int_0^{\infty} e^{-nt}[u(z_0+z_1,t)-e^{nt}(z_0+z_1)-e^{nt}k_0h(v_0)-e^{nt}k_1h(v_1)]$$

$$\text{subject to} \quad z_0\leq k_0,\ z_1\leq k_1$$

$$\dot{k}_0=k_0(v_0-\delta-n)$$

$$\dot{k}_1=k_1(v_1-\delta-n)$$

$$k_1(0),\ k_0(0) \text{ given.} \tag{3.14}$$

This problem has a well-defined "pseudo-steady-state" solution with both firms' output equal to capacity growing at the rate n, with a ratio $K(t)e^{-nt}=k^*=z^*$ and a constant investment/capacity ratio $I_i(t)/K_i(t)=v^*$. In the case of a dominant public firm the investment policy is now amended to

$$\dot{k}_0 = r(k_1^*-z_1)+F(2z^*-z_1-k_0) \tag{3.15}$$

The analysis and results proceed exactly as before, except all statements are phrased in terms of growth-adjusted variables. The symmetry of firms is important in this analysis of course, in that we need a "long-run" characterized by equal growth rates for both firms.

Throughout this section we have assumed that the public firm's investment policy depends upon the capacity, or output decisions of the private firm and not on the investment decision of the private firm. If it were possible for the public firm to monitor $I_1(t)$ at each instant and react to it, it would be possible to obtain not only long-run optimality but also full dynamic optimality. The public firm would announce a policy

$$I_0(t) = I_0^*(t)+I_1^*(t)-I_1(t), \qquad (3.16)$$

where $I_i^*(t)$, i=0,1 denote the socially optimal investment trajectories. Given (3.16) it is clear that $K_0(t)+K_1(t) = K_0^*(t)+K_1^*(t)$ at each t; thus the private firm becomes an intertemporal price taker and the best it can do is to choose $I_1(t)=I^*(t)$.

The problem with (3.16) is two-fold. First, it is an extremely complex policy to announce to the private firm as it is time dependent. The second problem is of an informational nature. The capital stocks may be easier variables for the public firm to observe than the investment flows, due for example to the costs of acquiring information instantaneously. It is equivalent in a period analysis to assuming that the ex ante investment plans of the private firms are not monitored, but that the ex post results of these decisions are monitored. Because of this lag in information, we do not expect to obtain the optimal distribution of capital and investment at each point of time. The best that can be hoped for is some longer run notion of optimality. Notice that in an open-ended model such as the one considered in this section the inability of the public firm to ensure full dynamic optimality hinges not only on the fixed nature of its capacity in the short-run, but also on information lags which limit its ability to react to actions taken in the short run by the private firm.

4. Multiple Private Firms

If there is more than one private firm in the industry there is a significant complication in that the private firm must account not only for the effect of its actions on the public firm, but must also account for how its actions will affect the other private firms and the consequent feedback effects. Since only the public firm is assumed to announce a given strategy as in (3.4), the private firms will be involved in a non zero-sum intertemporal game. We shall assume, as is traditional in oligopoly theory, that firms play this game in a non-cooperative fashion. If they cooperated to maximize joint profits we would be back to the two firm case. The appropriate equilibrium concept in such games, however, is far from settled; in particular, even if some Nash definition of equilibrium is adopted, what constitutes an acceptable strategy space is an open question. In the literature there appear to have been three suggestions as to appropriate strategy spaces. We must distinguish between a "move" at time t, which is simply the choice of a decision variable made by a player at time t, and a "strategy" which is some rule for making moves at each t. In some of the literature, in particular that on supergames, a move at time t is allowed to depend on all moves made in the previous periods; it is not allowed to depend on moves made in the future.[9] Conversely, in the literature on differential games an "open-loop" strategy is one which is chosen at the beginning of the game and is allowed to depend on the entire future course of the game; more precisely the entire future sequence of moves by all other players. A third notion of a strategy is one motivated by a dynamic-programming type argument. This is the "closed-loop" strategy of differential game theory and is discussed extensively in Starr and Ho (1969). As in much dynamic theory we distinguish between the "state" of the game, x_t, and admissable controls or "moves", u_t, at each time t. A set of moves by all players at t results (usually) in a change in the state of the game via some exogenously given transition rule, say $x_{t+1}=T(x_t,u_t)$. A closed-loop equilibrium is derived via the usual dynamic programming argument of backward induction. Thus at each instant t_0 a set of moves is chosen for that period assuming the relevant payoff includes only that period and all future periods; furthermore the payoff in future periods is assumed to be the Nash payoff. The closed-loop equilibrium strategies in period t are simply the ordinary Nash equilibrium strategies of a static game with respect to these payoffs in period t. Under reasonable regularity conditions a closed-loop strategy can be written as a function of the "state" of the game and time, say $u_t=\psi(x_t,t)$. Under conditions of stationarity in an infinite-horizon game closed-loop strategies will depend only on the state, $u_t=\psi(x_t)$. An important property of the "closed-loop" Nash equilibrium is that it is stable against revision; that is as time proceeds players do not have an incentive to change their strategies. This is not true of the other proposed Nash equilibrium. In this section we shall first consider the closed-loop Nash equilibrium as the equilibrium concept. This is the one which seems to have been used most commonly in the economics literature.[10]

Assume there are n+1 firms; n private firms, $i=1,\cdots,n$ and one public firm, i=0. The assumptions are as in the previous section, except (3.4) now becomes

$$\dot{K}_0 = r(K^T-\Sigma_{i=1}^n K_i)+F(Q^T-Q). \qquad (4.1)$$

$\{K_i^*\}$ and Q^* denote the appropriate long-run socially optimal capacities and industry output. Throughout this section we will ignore the excess capacity problem and assume $Q_i=K_i$ for all i; this could be established as in section 3 as a consequence of the analysis (at least in the long-run). Let $\psi_i(x)$ denote the capacity investment strategy for the ith firm, i=1,⋯,n where $\psi_i(\cdot)$ is a function depending on the state of the game, $x=(K_0,K_1,\cdots,K_n)$, and $I_i(t)=\psi_i[x(t)]$.

Starr and Ho (1969) have given a set of necessary conditions for $\{\psi_1,\cdots,\psi_n\}$ to be a set of closed-loop Nash equilibrium strategies. Define the (current-value) Hamiltonian for the jth player as a function

$$H_j(x,u,\lambda_j) = K_jD(Q)-cK_j-G(I_j)+\Sigma_{i=1}^n\lambda_{ji}(I_j-\delta K_j)$$
$$+\lambda_{j0}\dot{M}(x),\ j=1,\cdots,n, \qquad (4.2)$$

where $u=(I_1,\cdots,I_n)$, $\lambda_j=(\lambda_{j0},\lambda_{j1},\cdots,\lambda_{jn})$, and M(x) equals (4.1).

Necessary conditions for $\{\psi_1,\cdots,\psi_n\}$ to be closed-loop Nash equilibrium strategies are that there exist a set of co-state variables λ_j that satisfy the following equations:

$$\dot{K}_j = I_j-\delta K_j,\ \dot{K}_0 = M(x),\ j=1,\cdots,n. \qquad (4.3.1)$$

$$\dot{\lambda}_{ji}=(r+\delta)\lambda_{ji}-K_jD'(Q)-\lambda_{j0}\frac{\partial M}{\partial K_i} - \Sigma_{\substack{k=1\\k\neq j}}^n \lambda_{jk}\frac{\partial\psi_k(x)}{\partial K_i} \qquad (4.3.2)$$

$i\epsilon\{1,\cdots,n\}$,
$i\neq j$,

$$\dot{\lambda}_{jj}=(r+\delta)\lambda_{jj}-K_jD'(Q)-[D(Q)-c]-\lambda_{j0}\frac{\partial M}{\partial K_i}-\Sigma_{\substack{k=1\\k\neq j}}^{n}\lambda_{jk}\frac{\partial\psi_k(x)}{\partial K_j} \quad (4.3.3)$$

$$\dot{\lambda}_{j0}=r\lambda_{j0}-K_jD'(Q)-\lambda_{j0}\frac{\partial M}{\partial K_0}-\Sigma_{\substack{k=1\\k\neq j}}^{n}\lambda_{jk}\frac{\partial\psi_k(x)}{\partial K_0} \quad (4.3.4)$$

$\tilde{I}_j^{*}=\psi_j(x)$ is the solution to

$$\max_{I_j} H(x,I_1',\cdots,I_j,\cdots,I_n',\lambda). \quad (4.3.5)$$

The above conditions hold for all private firms $j=1,\cdots,n$. Note that I_k', $k\neq j$ is treated as a parameter in (4.3.5) and equal to $\psi_k(x)$.

λ_{jk} is a co-state variable which measures the influence of the kth state variable on the jth firm. The term $\lambda_{jk}\partial\psi_k(x)/\partial K_i\equiv\partial H_j/\partial I_k\cdot\partial I_k/\partial K_i$ is the interaction between the ith state variable and kth state variable due to the interdependence between firms.

We will assume at least one equilibrium set of strategies exists and furthermore that all equilibrium strategies converge to an interior steady-state.[11]

For any private firm j the state of the game $x=(K_0,\cdots,K_n)$ can be adequately described by the pair $(K_j,\Sigma_{i\neq j}\ K_i)$; this is because these two numbers, firm j's own capacity and the capacity of all other firms, are sufficient to determine the payoff in any state x to firm j. Consequently $\psi_j(x)$ can be written as $\hat{\psi}_j(K_j,\Sigma_{i\neq j}K_i)$, which implies $\partial\psi_j/\partial K_i=\partial\psi_j/\partial K_m$, all $i,m\neq j$. Using this result, the steady-state equations (4.3.3) and (4.3.4), and noting that $\partial M/\partial K_j=-r+\partial M/\partial K_0$, we get the condition $(r+\delta)\lambda_{jj}=D(Q)-c$, or from (4.3.6)

$$(r+\delta)G_j'(\delta K_j)=D(Q)-c \qquad j=1,\cdots,n. \quad (4.4)$$

In a steady-state (4.1) becomes

$$K^T-\Sigma_{j=1}^{n}K_j+F(Q^T-Q)=0. \tag{4.5}$$

As in the previous section it is possible to choose (Q^T,K^T) and function $F(\cdot)$ such that the unique solution to (4.4) and (4.5) is $Q=Q^*$ and $K_j=K_j^*$ all $j=1,\cdots,n$. Thus the basic result of section 3 in the presence of multiple private firms behaving non-cooperatively remains provided a closed-loop Nash equilibrium is the appropriate strategy and a steady-state solution is the asymptotic outcome of the dynamic oligopoly game.

In what way can the above assumptions be relaxed? The existence of a steady-state as a long-run equilibrium is again crucial. In the short-run the previously mentioned inefficiencies can arise. It turns out that the closed-loop notion of equilibrium is not crucial. It is easily shown that if the Nash equilibrium trajectory is determined by firms adopting open-loop strategies relative to (4.1) then provided the equilibrium trajectory converges, it necessarily converges to the socially optimal steady state for an appropriately chosen reaction function (4.1).[12] This is in some sense surprising since it is well known open and closed loop equilibrium need not coincide. This may also be true here, but their asymptotic states necessarily coincide.

The results of this section are quite comforting. With two quite different notions of equilibrium in a dynamic non-cooperative game we have established that the long-run outcome of the game, given the dominant public firm adopts a policy as in (4.1), will coincide with the socially optimal steady-state.

5. Conclusion

The basic result of this paper is that long-run welfare optimality can be achieved in a potentially oligopolistic situation if a public firm which is dominant in the industry can suitably punish restrictive and inefficient output/investment decisions by private firms in the industry. By a long-run welfare optimum we mean a situation in which all firms are in a stationary state with long-run marginal cost equal to industry price. While long-run optimality can be ensured, short-run or dynamic non-steady-state welfare losses will in general occur due to the exercise of market power by private firms. This market power exists as a consequence of the short-run capacity constraints placed on the public firm which limit its ability to carry out retaliatory policies.

The basic decision rule proposed for the public firm was an investment in capacity rule stating that net additions to its capacity should be made as an increasing function of deviations of the private firm's output from its target capacity and of deviations of the industry output from target output. A crucial condition to ensure long-run optimality is that the rule for additions to public firm capacity, $\dot{K}_0$, satisfy the condition

$$\frac{\partial \dot{K}_0}{\partial q_1} = -r + \frac{\partial \dot{K}_0}{\partial q_0} .$$

Thus a unit decrease in private firm output must yield an increment to new capacity in the public firm which is equal to r, plus the increment to public firm capacity which would occur if the public firm itself had cut back output equal to capacity by one unit. Thus the speed of reaction by the public firm to private firm actions must be faster, by a factor equal to the discount rate, than its response to its own output decision. The effect of this adjustment is that the private firm faces a "long-run" demand curve which is perfectly elastic, or to be more precise, the long-run value of an additional unit of output is simply price minus long-run marginal cost. To see this, consider any interior steady-state. Then by the envelope result

$$\frac{\partial \pi_1}{\partial K_0} = \int_0^\infty e^{-rt}[K_1 D'(Q) - \lambda_0 F']$$

and

$$\frac{\partial \pi_1}{\partial K_1} = \int_0^\infty e^{-rt}[D(Q) - c - \delta\lambda_1 + K_1 D'(Q) - r\lambda_0 - \lambda_0 F'].$$

But $\partial \pi_1/\partial K_0 = \lambda_0$, so we have

$$\frac{\partial \pi_1}{\partial K_1} = \int_0^\infty e^{-rt}[D(Q) - c - \delta\lambda_1] + \frac{\partial \pi_1}{\partial K_0} - \lambda_0$$

$$= \int_0^\infty e^{-rt}[D(Q) - c - \delta\lambda_1]$$

or, $\partial \pi/\partial K_1 = (D(Q)-c)(r+\delta) = G'(\delta K)$; i.e. price equal long-run marginal cost.

The basic difficulty with the rule proposed is that the public firm could incur quite high costs during adjustment periods. Thus it is possible that the present-value of the public firm's revenues minus costs at t=0 could be negative, particularly if $\dot{K}_0$ is extremely large in early periods and the discount rate is very low. This might be infeasible if the public firm must operate with non-negative present value. Secondly, the usual informational difficulties arise in that the public firm must have knowledge of demand and cost conditions, including the costs of private firms. These problems do not seem insurmountable though, and in any case do not seem any worse than the usual problems one runs into with schemes involving public intervention. They clearly provide considerable scope for further research.

In summary then the dominant public firm scheme can provide a means of eliminating long-run welfare losses from oligopolistic pricing, output and investment policies. Finally we note that if the social welfare function were to weight consumer surplus more heavily than producer surplus, rather than equally as in this analysis, then the benefits of the scheme may be much greater, since it transfers real income from producers to consumers, relative to a no-intervention policy.

Footnotes

1. There are significant examples of such companies in almost all western economies. Some examples are: British Leyland and British Petroleum in the U.K.; Air Canada and Petrocan in Canada; Renault in France; Lufthansa in Germany; Quantas Airlines in Australia. Some case studies of industries in which a public firm competes with a few private firms are Davies (1971), Martin (1959) and Sheahan (1960). More extensive documentation of such firms and industries is available in Harris and Wiens (1979).
2. A role for public intervention in such cases is clear: to eliminate the welfare loss from inefficient industry production and restrictive output.
3. Throughout the analysis we take the number of firms as fixed. This could be justified by assuming there are substantial entry barriers; eg., the technology is such that all firms have at least some region of decreasing costs, and hence the number of firms in the market is limited by demand conditions.
4. The analysis assumes that both firms are currently in the industry; thus the entry of either is not an issue. The use of capacity as an entry deterrent in the traditional private-private oligopoly game has been the subject of some recent interesting analysis. See Dixit (1979), Eaton and Lipsey (1976) Spence (1977), and Wenders (1971).
5. The operating cost function could be generalized to depend upon capacity, say $c_i(q_i,K_i)$. It would not change the results to follow in any way provided c_i were convex. A realistic generalization would be to include fixed costs in the analysis. Harris (1978) provides a static treatment with fixed

costs, but in a dynamic context the analysis becomes considerably more difficult; the results of Davidson and Harris (1979) might usefully be applied however.

6. Rockafellar (1970) contains a statement of the appropriate generalization of the maximum theorem for non-differentiable Hamiltonians. In particular (3.7) follows as the interval $[A_0,B_1]$ is the sub-differential of the Hamiltonian with respect to K_1, when $\tilde{q}_1=K_1$.
7. It is not possible in a control problem which is stationary except for exponential discounting to have optimal trajectories which are closed orbits. For a proof see Davidson and Harris (1979), appendix 1.a.
8. The only analysis in private oligopoly games we are aware of which takes into account both growth in demand and dynamic entry considerations is the recent paper by Spence (1979).
9. A survey of the literature on supergames is contained in Friedman (1977).
10. Clemhout and Wan (1979) is a recent survey of the applications of differential game theory to economics. As they point out most applications have used the closed loop notion of Nash equilibrium.
11. Existence of Nash equilibrium is as troublesome in differential games as in ordinary static game theory. A proper treatment of the existence question is beyond the scope of this paper. The assumption that equilibrium, strategies converge to a steady-state seems reasonable; it precludes any firm from running down its capacity to zero, becoming infinitely large, or following a regular cycle of expansion and contraction. Unfortunately to establish stability one needs to know the $\{\psi_j\}$ functions. As Starr and Ho (1969) point out, it is extremely difficult, except in the simplest cases, to solve for these.

12. As shown in Case (1969) the necessary conditions for an open-loop Nash equilibrium are the same as in (4.3) except that in each case the terms $\partial\psi_k/\partial K_j$ do not appear. Recall that an open-loop Nash equilibrium is one in which all players choose the entire sequence of their controls $\{I_j(t)\}_{t=0}^{\infty}$ given the control sequences chosen by all other players.

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